




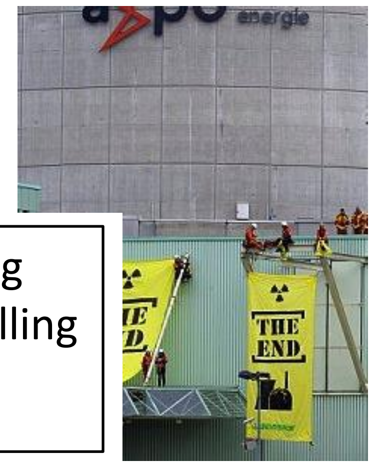

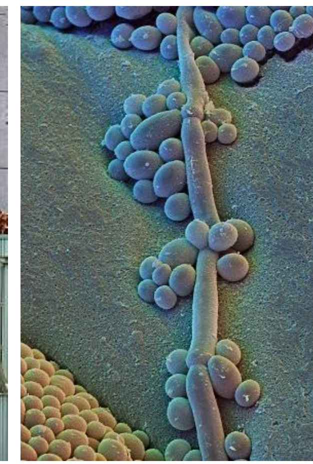

The Sequential Refined Conditioning method: addressing under- and overspecification of EJ in dependence modelling

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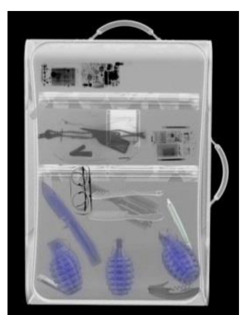


Motivation(s) of our modelling/elicitation method

Why use the sequential refined conditioning method?

- Address the ***underspecification*** issue of assessed dependence models*:
- Underspecification means that we have not elicited enough information for modelling a unique distribution as various alternatives are compatible with the given (partial) information

It is desirable that the resulting joint distribution is unique and is only based on experts' judgements, i.e. no unspecified assumptions

- Proposed solution: modelling non-assessed parts of distribution as minimally informative

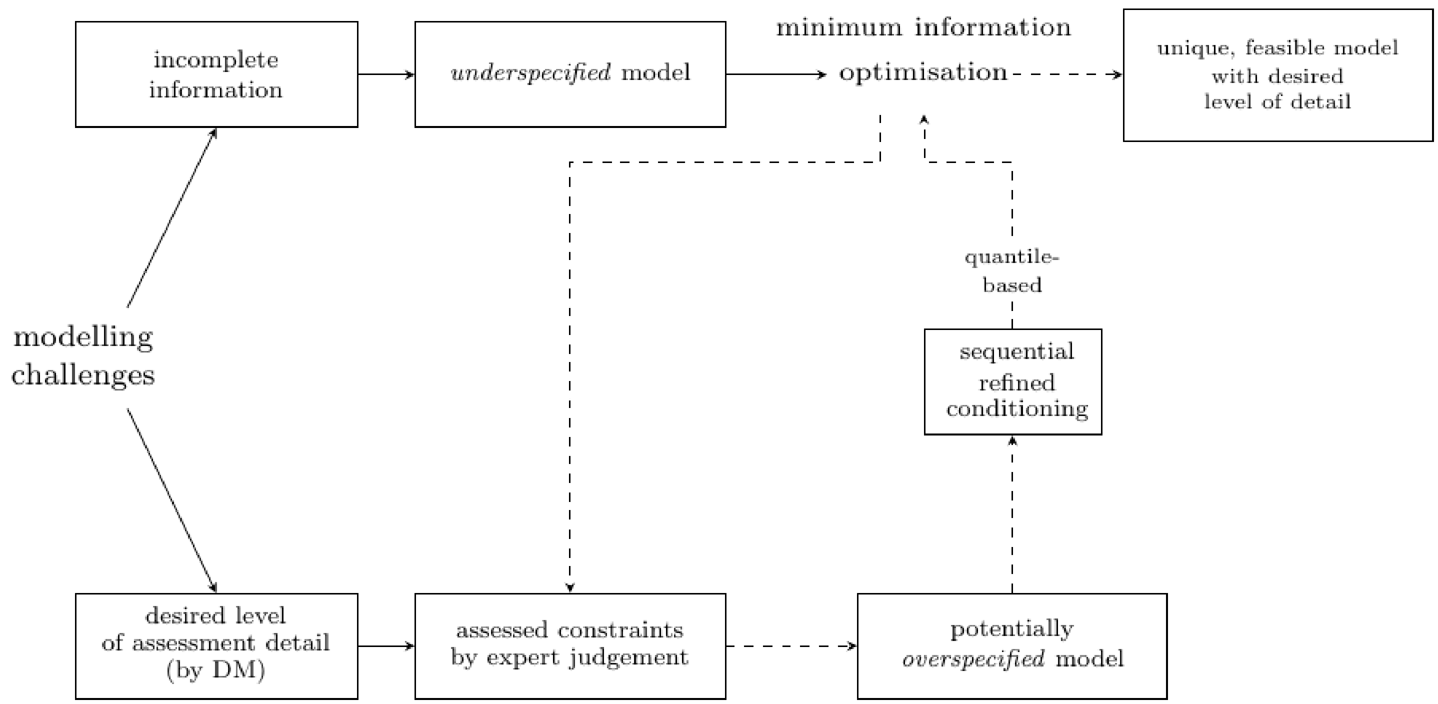
- Address the ***overspecification*** issue of assessed dependence models*:
- For overspecification, an expert's assessments about related parts of a distribution are contradictory and infeasible; potentially occurring due to an increased cognitive complexity for experts when assessing a variety of detailed, related distribution features

It is desirable that the assessments exhibit a low cognitive complexity for experts despite allowing for flexibility of the assessed parts and level of detail of the distribution

- Proposed solution: only ever eliciting single conditioning sets, explicit guidance on feasible ranges

**note: we consider non-parametric dependence models, under- and overspecification might also occur in parametric settings*

Modelling context of the SRC method



Addressing overspecification:

- Proposing a sequential elicitation procedure that gives explicit guidance on feasible assessments (in any part and level of detail of the joint distribution) and only ever elicits single conditioning sets:

    1. initial four step procedure (only marginals are specified at this point)
    2. further assessing *within given area*
    3. further assessing *newly given area*

SRC: initial elicitation sequence (1/4)

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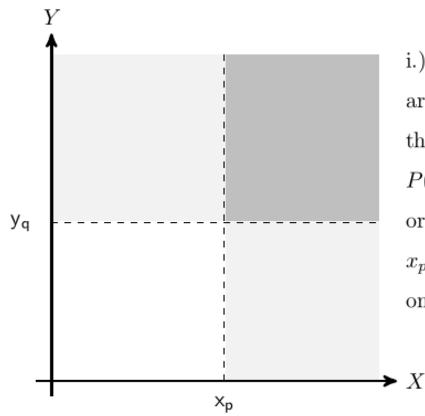
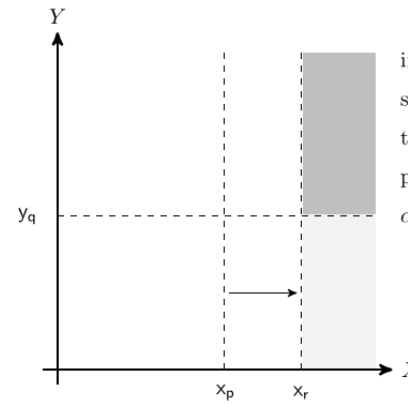
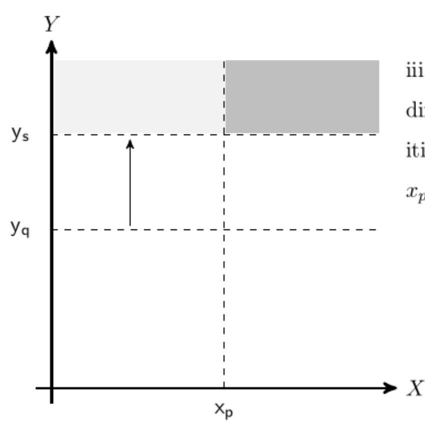


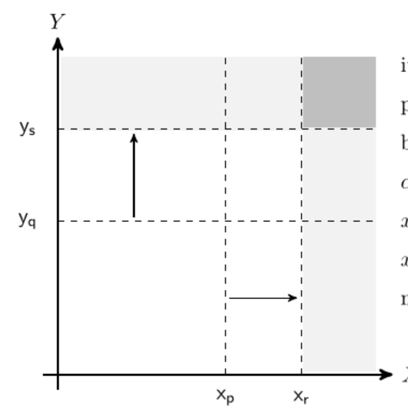
i.) we specify the area that is then further refined, $\alpha_1 = P(Y > y_q | X > x_p)$ or $\alpha_1 = P(X > x_p | Y > y_q)$ depending on $\min[1-p, 1-q]$.



ii.) we refine the assessed α_1 through asking the expert for a smaller part of this area through $\alpha_2 = P(Y > y_q | X > x_r)$.



iii.) we refine α_1 with a different focus by eliciting $\alpha_3 = P(X > x_p | Y > y_s)$.



iv.) we refine the two previous assessments by asking an expert for $\alpha_4 = P(Y > y_s | X > x_r)$ or $\alpha_4 = P(X > x_r | Y > y_s)$ depending on $\min[1-r, 1-s]$.

SRC: initial elicitation sequence (2/4)

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- Feasible ranges are given by:

$$0 \leq \alpha_1 \leq 1$$

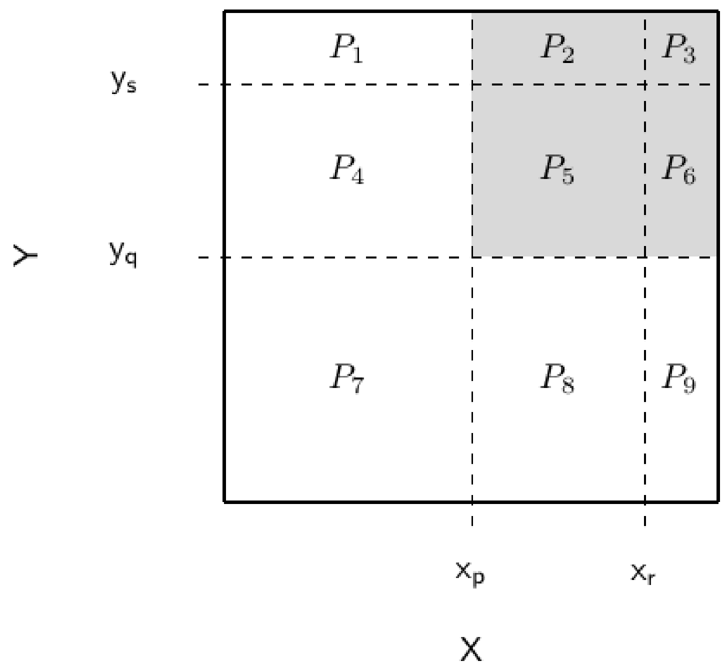
$$0 \leq \alpha_2 \leq \begin{cases} \min\left[1, \dfrac{(1-p)\alpha_1}{1-r}\right] if\ p \geq q \\ \min\left[1, \dfrac{(1-q)\alpha_1}{1-r}\right] if\ q \geq p \end{cases}$$

$$0 \leq \alpha_3 \leq \begin{cases} \min\left[1, \dfrac{(1-p)\alpha_1}{1-s}\right] if\ p \geq q \\ \min\left[1, \dfrac{(1-q)\alpha_1}{1-s}\right] if\ q \geq p \end{cases}$$

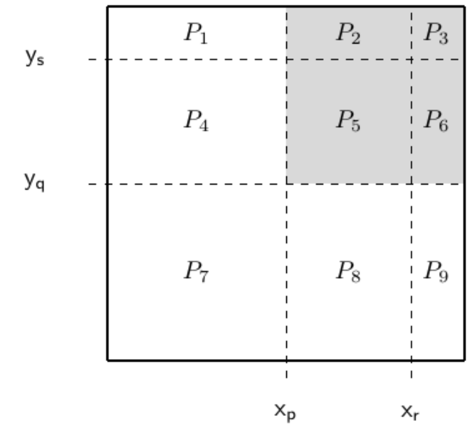
$$0\ \max[0, \frac{-\min[1-p, 1-q]\alpha_1 + (1-r)\alpha_2 + (1-s)\alpha_3}{\min[1-r, 1-s]}] \leq \alpha_4 \leq \begin{cases} \min\left[\alpha_2, \dfrac{(1-s)\alpha_3}{1-r}\right] if\ r \geq s \\ \min\left[\alpha_3, \dfrac{(1-r)\alpha_2}{1-s}\right] if\ s \geq r \end{cases}$$

SRC: initial elicitation sequence (3/4)

- After the initial elicitation sequence (all four steps; we can always stop before), the joint distribution is given as below:



SRC: initial elicitation sequence (4/4)



- Resulting probability masses are given by:

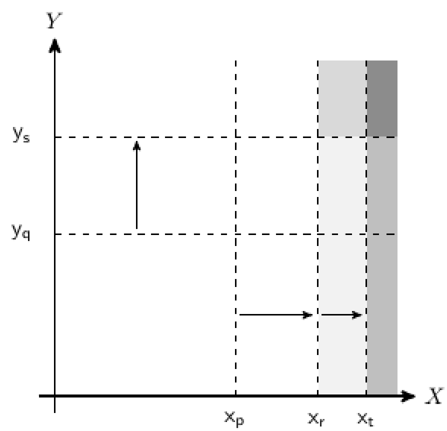
$$P_3 = \min[1-r, 1-s]\alpha_4$$

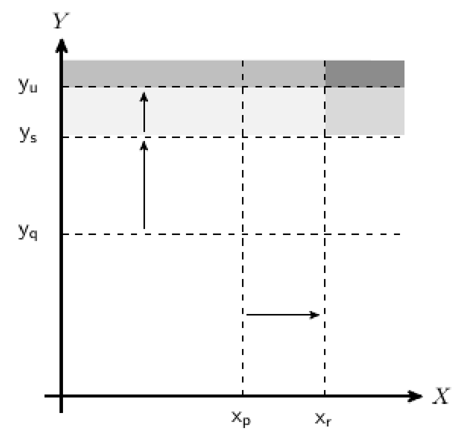
$$P_6 = (1-\mathrm{r})\alpha_2 - \min[1-r, 1-s]\alpha_4$$

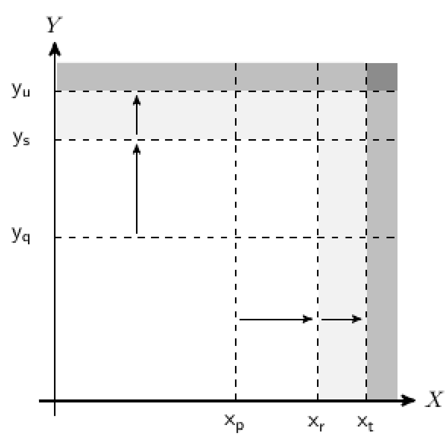
$$P_2 = (1-\mathrm{s})\alpha_3 - \min[1-r, 1-s]\alpha_4$$

$$P_5 = \min[1-p, 1-q]\alpha_1 - (1-r)\alpha_2 - (1-\mathrm{s})\alpha_3 + \min[1-r, 1-s]\alpha_4$$

SRC: further assessing *within given area* (1/4)







v.) $\alpha_5 = P(Y > y_s | X > x_t)$,
vi.) $\alpha_6 = P(X > x_r | Y > y_u)$
and vii.) $\alpha_7 = P(Y > y_u | X > x_t)$ or $\alpha_7 = P(Y > y_u | X > x_t)$ (clockwise): refining further the previously assessed area of P_4.

SRC: further assessing *within given area* (2/4)

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- Feasible ranges are given by:

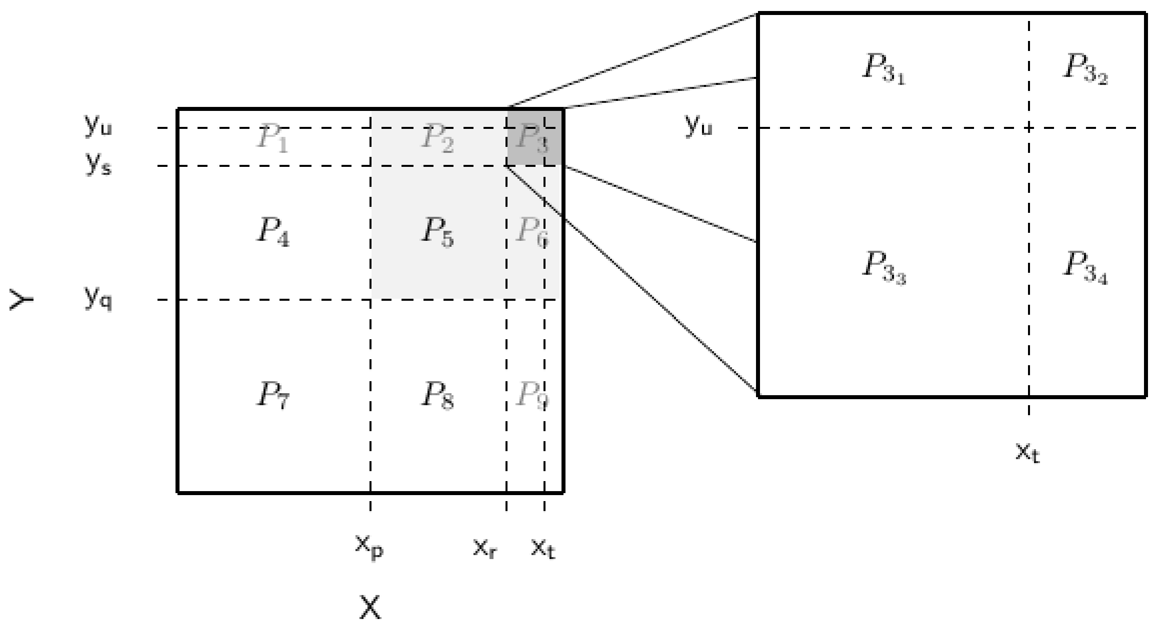
$$0 \leq \alpha_5 \leq \frac{\min[1-r, 1-s]\alpha_4}{1-t}$$

$$0 \leq \alpha_6 \leq \frac{\min[1-r, 1-s]\alpha_4}{1-u}$$

$$\max[0, \frac{-\min[1-r, 1-s]\alpha_4 + (1-t)\alpha_5 + (1-u)\alpha_6}{\min[1-t, 1-u]}] \leq \alpha_7 \leq \begin{cases} \min\left[\alpha_5, \frac{(1-u)\alpha_6}{1-t}\right] if\ t \geq u \\ \min\left[\alpha_6, \frac{(1-t)\alpha_5}{1-u}\right] if\ u \geq t \end{cases}$$

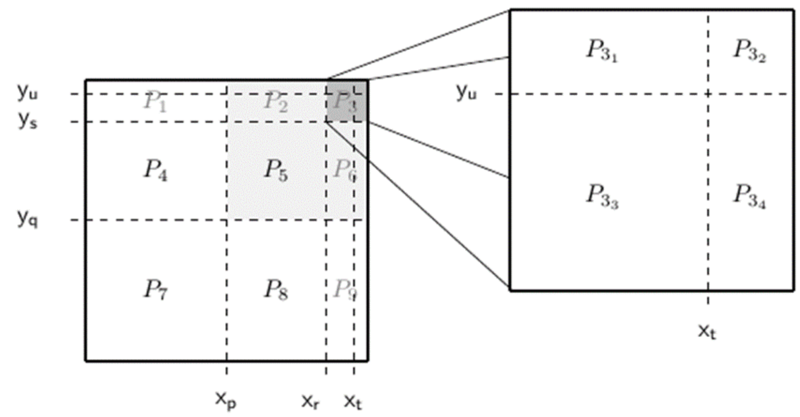
SRC: further assessing *within given area* (3/4)

- After further within a given area, the joint distribution is given as below:



SRC: further assessing *within given area* (4/4)

- Resulting probability masses are given by:



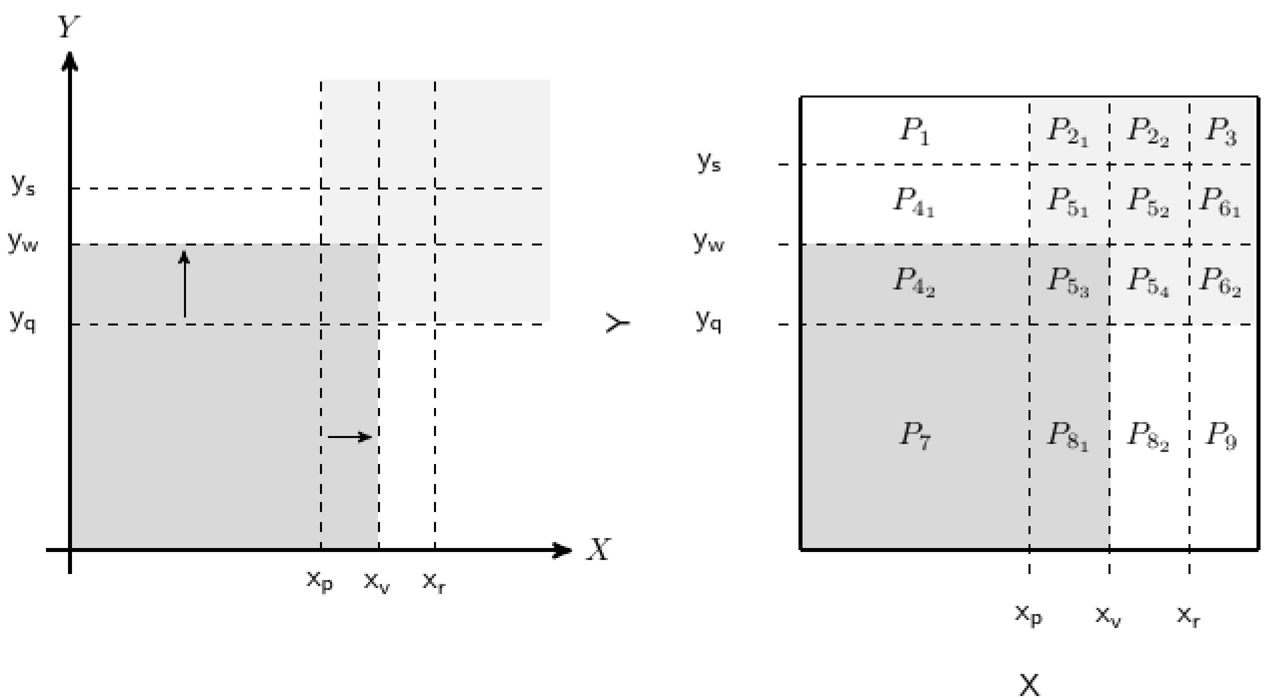
$$P_{3_2} = \min[1-t, 1-u]\alpha_7$$

$$P_{3_1} = (1-\mathrm{t})\alpha_5 - \min[1-t, 1-u]\alpha_7$$

$$P_{3_4} = (1-\mathrm{u})\alpha_6 - \min[1-t, 1-u]\alpha_7$$

$$P_{3_3} = \min[1-r, 1-s]\alpha_4 - (1-t)\alpha_5 - (1-\mathrm{u})\alpha_6 + \min[1-t, 1-u]\alpha_7$$

SRC: further assessing *newly given area*



Addressing underspecification:

- Proposing a minimum information solution:
- Formally, we aim for modelling dependence through distribution which is chosen to have minimum information (Kullback-Leibler divergence (Kullback and Leibler, 1951)) with respect to the independent uniform distribution with the same marginals given a finite number of constraints

Illustrative example (1/3):
Assessing spatial dependence of terrorism risk

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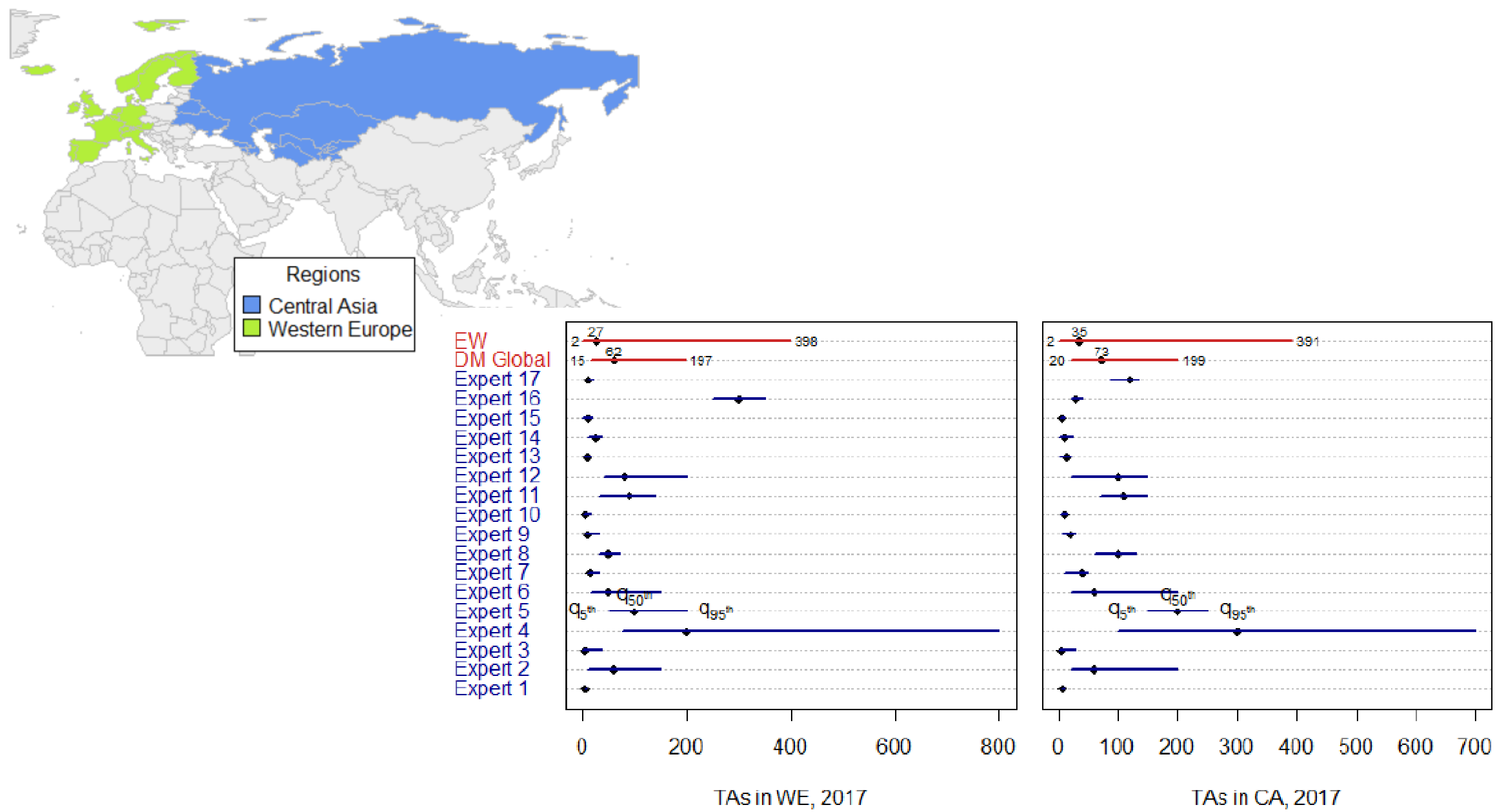
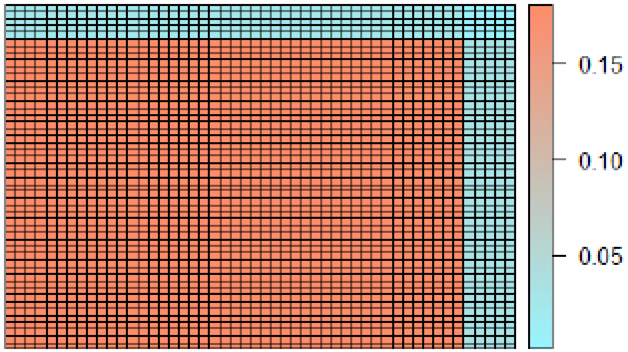
- Experts were insurance underwriters and professionals of related service providers
- Elicitation of marginal distribution through the Classical model (Cooke, 1991)
- Terrorism risk came to the attention of insurers after 9/11
  - Before, often covered as an unnamed peril under an all-risk commercial and home owners coverage for property and contents (e.g. in US)
  - More generally, the worst 15 terrorist attacks in terms of number of casualties have occurred since 1982
  - Mathematically, problem of terrorism in terms of frequency-severity relationships can be described by a power law, i.e. attack severities order of magnitude larger than the mean might not be unusual
- Terrorism risk poses particular challenges due to intelligent adversaries
  - Spatial dependence evoked from attackers through globally and locally active terrorist groups; such foci are often due to motivations, followed ideologies, and structure of groups
    - E.g. some groups are hierarchically structured, others work as satellite cells which has an effect on counter-terrorism measures applicable
  - Spatial dependence also determined by defender's vulnerabilities

Illustrative example (2/3):
Assessing spatial dependence of terrorism risk



Illustrative example (3/3):
Assessing spatial dependence of terrorism risk

α_i	Framing ”Given that we observe [...]”	Conditional Probability	Assessment
α_1	”[...] more than **73** terrorist attacks in CA, what is your probability that we observe more than **62** terrorist attacks in WE?”	$P(Y > y_{0.5} \| X > x_{0.5})$	**0.5**
α_2	”[...] more than **199** terrorist attacks in CA, what is your probability that we observe more than **62** terrorist attacks in WE?”	$P(Y > y_{0.5} \| X > x_{0.95})$	**0.75**
α_3	”[...] more than **197** terrorist attacks in WE, what is your probability that we observe more than **73** terrorist attacks in CA?”	$P(X > x_{0.5} \| Y > y_{0.95})$	**0.9**
α_4	”[...] more than **199** terrorist attacks in CA, what is your probability that we observe more than **197** terrorist attacks in WE?”	$P(Y > y_{0.95} \| X > x_{0.95})$	**0.25**
α_5	”[...] more than **199** terrorist attacks in CA, what is your probability that we observe more than **225** terrorist attacks in WE?”	$P(Y > y_{0.99} \| X > x_{0.95})$	**0.1**
α_6	”[...] more than **225** terrorist attacks in WE, what is your probability that we observe more than **199** terrorist attacks in CA?”	$P(X > x_{0.95} \| Y > y_{0.99})$	**0.15**



Thank you for your attention.