Efficient Forecasting of Volcanic Ash Clouds

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Two basic questions addressed in this talk:

1. How does uncertainty affect forecasts of volcanic ash clouds?

2. How do we maximize the constraints observations place on model results and forecasts?

Brief Outline:

I. **Properties of volcanic ash clouds**
II. **Measurements of clouds input to models**
III. **Process of inversion and error propagation in model results**
IV. **Forecasting volcanic ash clouds**

Properties of volcanic ash clouds in the atmosphere

1. Ash mass is significant only at the eruption column source
   i.e. Costa et al, 2013

2. Transport in the atmosphere is passive, as mean settling time for
   mean size range in ash clouds is low (.036 km/hour @ 10 microns)
   and wind speeds are typically high (36 – 100 km/hr)

3. Simulations of ash clouds from eruption column source use wind solutions
   to obtain wind velocity and satellite data to constrain the source of
   volcanic ash clouds

4. As mass flux in a volcanic plume varies with altitude over time, wind shear
   changes the direction of the cloud streaming from the eruption column.

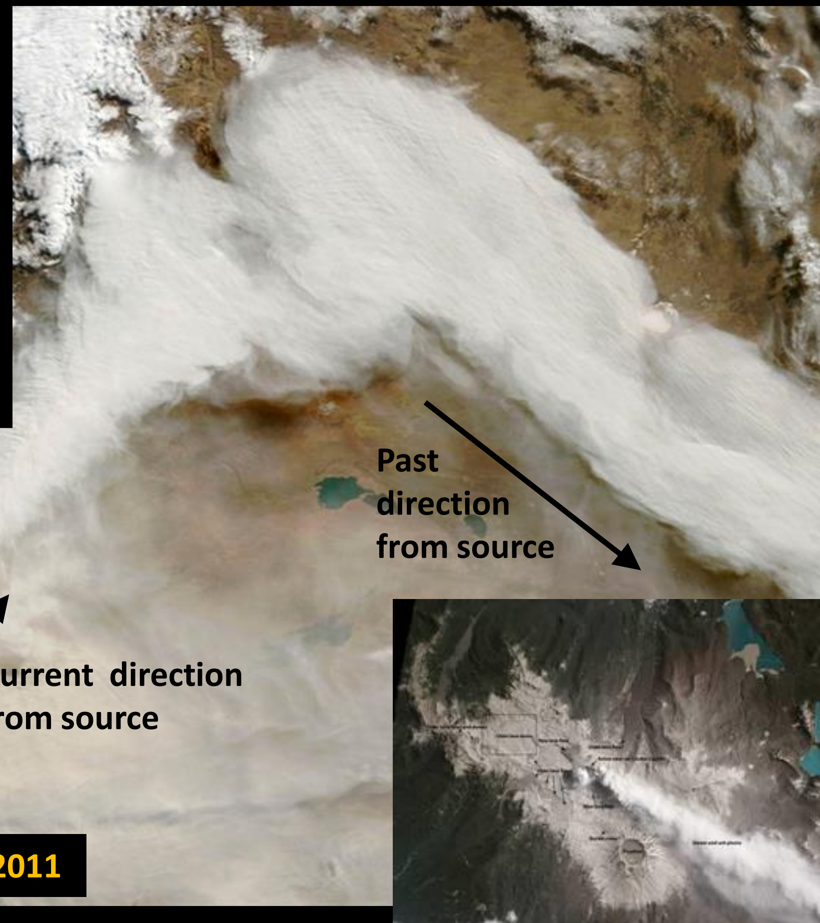
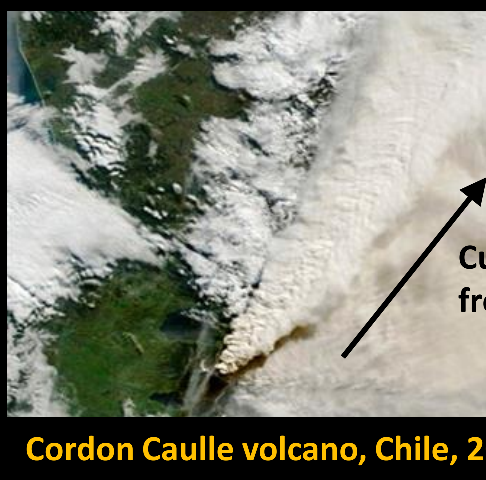
Errors in forecasting volcanic ash clouds are obtained from errors in :

- satellite data
- wind data
- transport models
- direct measurements of ash clouds

Example of cloud changes with wind shear

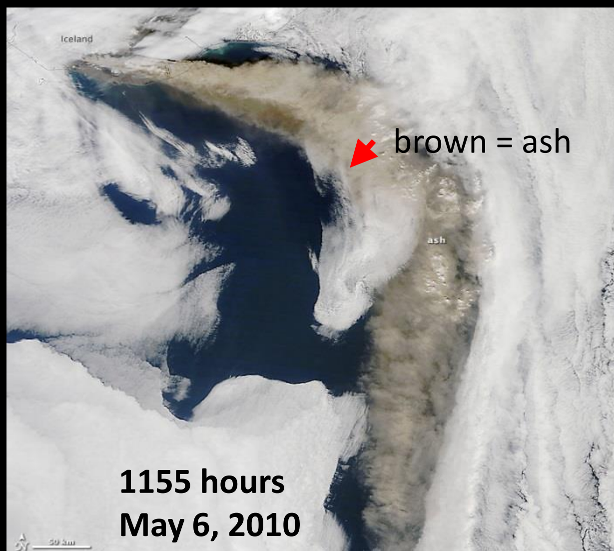
As either wind direction or ash altitude changes, the direction in which the ash cloud moves changes…

This information is used to train a computer to track and forecast an ash cloud

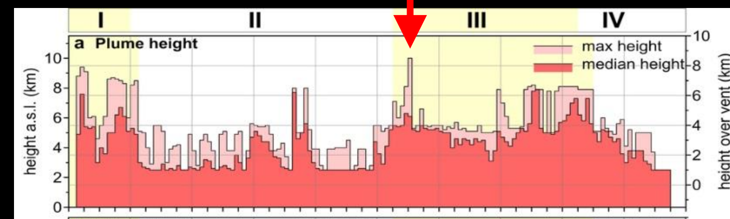



Cordon Caulle volcano, Chile, 2011

Errors in prior information of an ash cloud source



00h on May 6, 2010

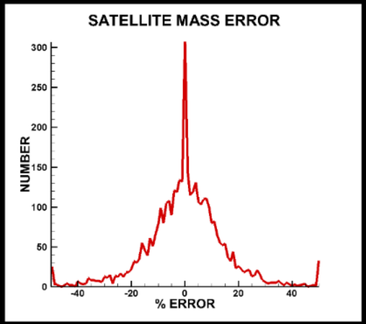


Gudmonsson et al., 2012; Arason et al., 2011

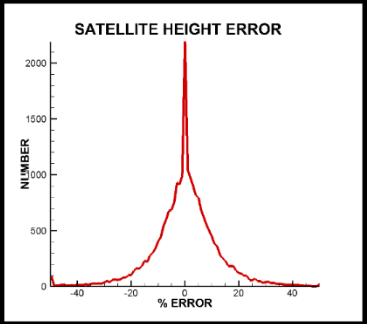
Compare observed plume in this satellite photo on the left with radar measurements above of plume height versus time. Which altitudes are supplying the ash cloud with ash?

Prior errors inherited from satellite data

- Satellite measurements of radiance used to infer the presence of an ash cloud and its altitude are resolved at many small areas above a potential ash cloud $\boldsymbol{\theta}$



Data vector sm
Mean value sm0



Data vector sh
Mean value sh0

- The errors in these data constrain model parameters by comparison of model clouds with actual satellite clouds

- The error in the prior satellite data is described with the prior distribution at right, with variance $1/\alpha$

$$p(\boldsymbol{\theta} \mid \alpha) = N(\boldsymbol{\theta} \mid \mathbf{0}, \alpha^{-1}\mathbf{I})$$

The error in winds and the transport of ash is given by the misfit between modeled ash clouds and actual satellite-determined clouds, and encapsulated into error parameter β

Likelihood of model parameters $\boldsymbol{\theta}$ given satellite data $\mathbf{D}(t^n)$ projected onto a discretized spherical grid determines the errors β in wind and transport

$$p(\mathrm{D}(\mathrm{t}^n)\,|\,\boldsymbol{\theta},\beta) = \exp\left[\frac{-\beta}{2}\sum_{j=1}^{j\max}\sum_{i=1}^{i\max}\left(M_{ij}\left(\boldsymbol{\theta},\mathrm{w}(t^n)\right)-sm0_{ij}\right)^2\left(M_{ij}\left(\boldsymbol{\theta},\mathrm{w}(t^n)\right)-sh0_{ij}\right)^2\right]$$

w = wind data
t^n = time at step n
$sm0_{ij}$ = mean ij satellite massload
$sh0_{ij}$ = mean ij satellite altitude
ij = longitude and latitude indices of each cell

Both α and β contribute to uncertainty in forecasting ash clouds

To resolve the uncertainty in satellite data to constrain and forecast volcanic ash clouds:

1. Use height and mass (cloud load) measurements of a growing ash cloud

The error of a likelihood estimation is contained in the Hessian of the misfit between model clouds and measured clouds

2. Determine the posterior distributions of model parameters

Use Bayes Theorem: For input parameters θ to a transport model *M* given ash cloud observation data *D*, the sum and product rules of probability give Bayes theorem:

$$P(\boldsymbol{\theta} \mid \mathbf{D}, M) = P(\mathbf{D} \mid \boldsymbol{\theta}, M)\, P(\boldsymbol{\theta} \mid M) / P(\mathbf{D} \mid M)$$

3. Use posterior distributions to weight model forecasts of ash clouds

The past success of each combination of inputs to a transport model in tracking an ash cloud determines its influence in forecasting

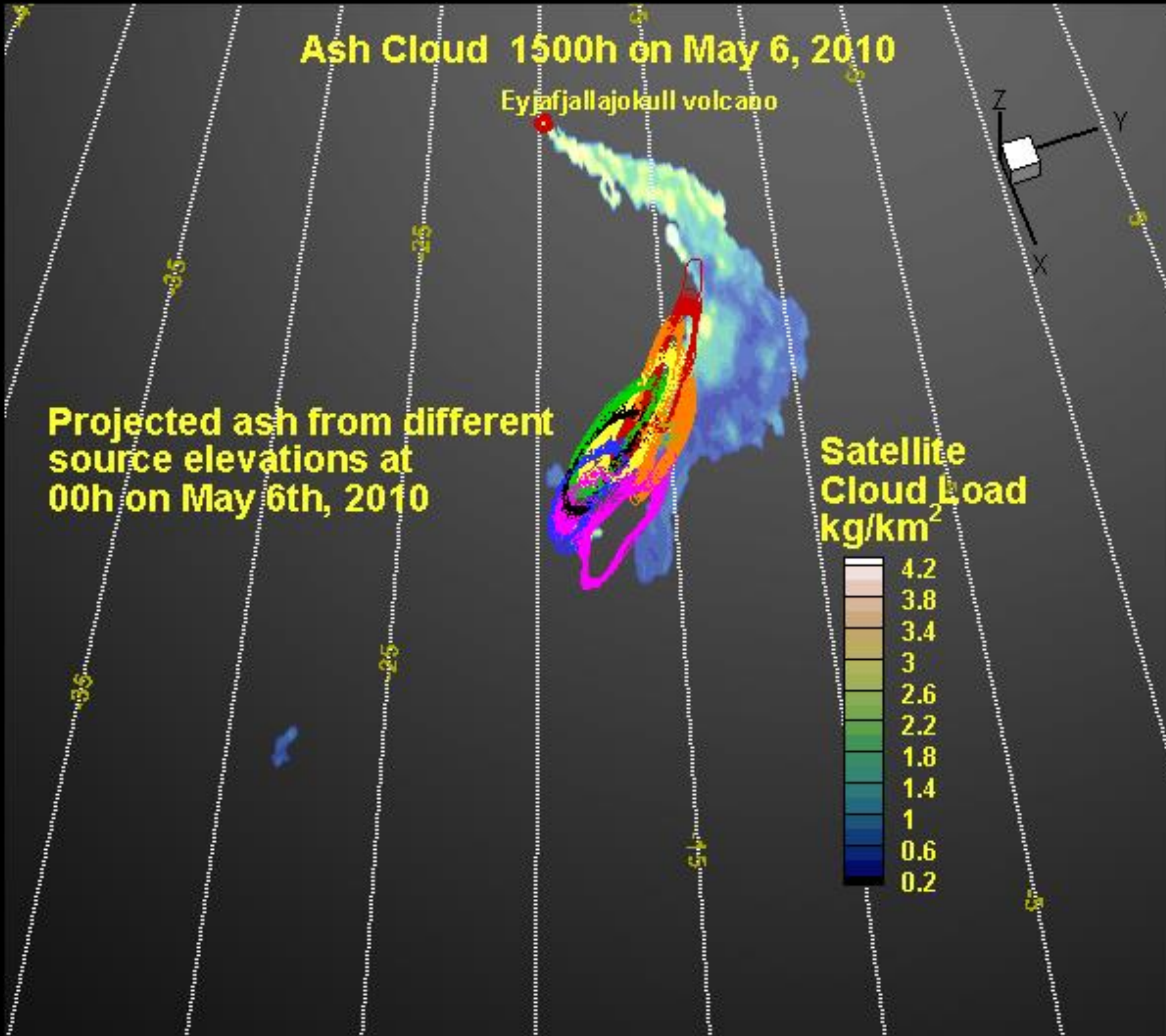
From Bayes theorem, the posterior distribution over the model parameters is proportional to the product of both the **prior** and **likelihood** distributions:

$$p(\boldsymbol{\theta} \mid D, \alpha, \beta) \propto p(D \mid \boldsymbol{\theta}, \beta)\, p(\boldsymbol{\theta} \mid \alpha)$$

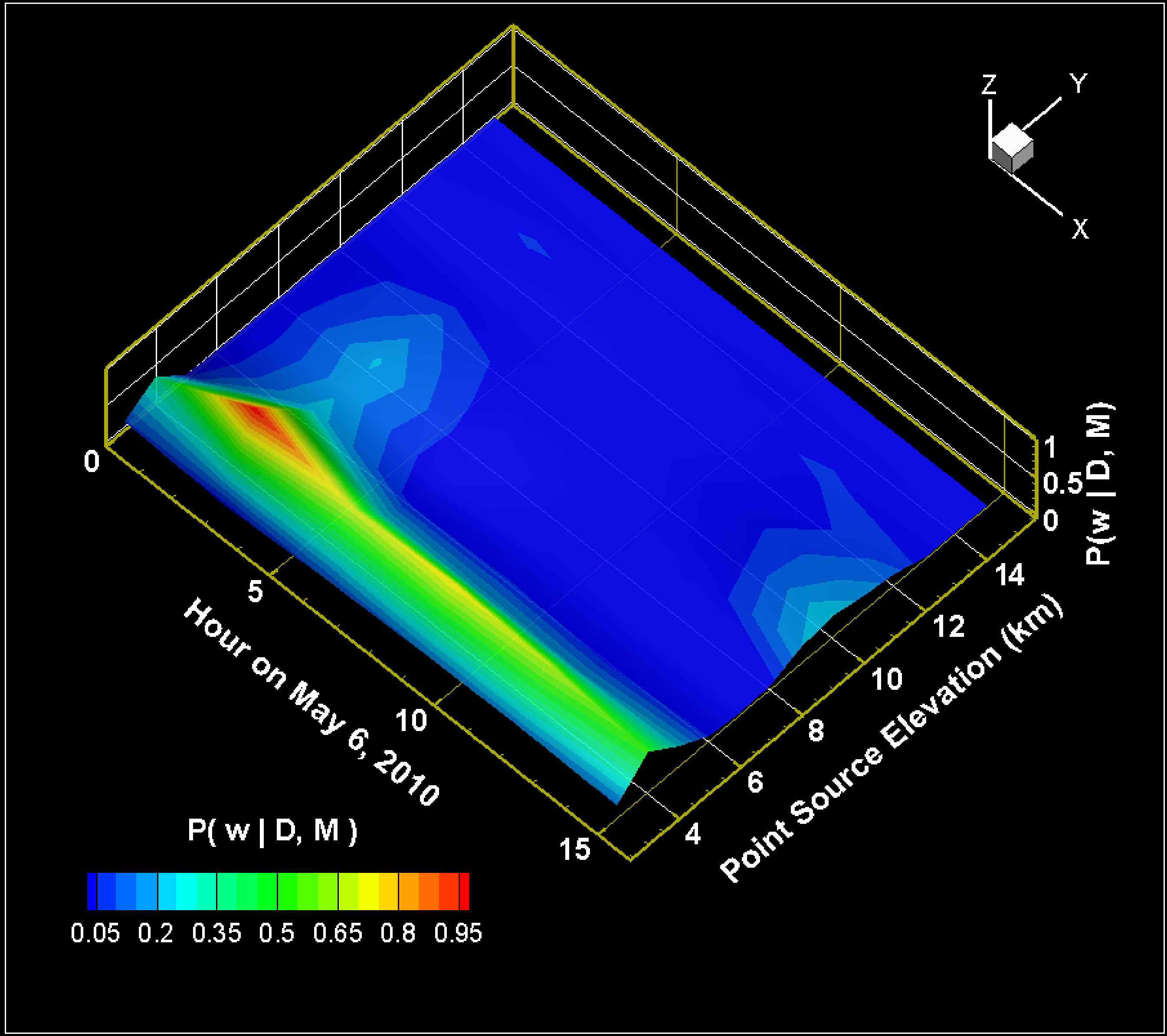
To be used in a forecast the posterior must be normalized by the integral over input parameters $\boldsymbol{\theta}$, which determines the **evidence** $p(D \mid \alpha, \beta)$ given by the data constraints:

$$p(D \mid \alpha, \beta) = \int p(D \mid \boldsymbol{\theta}, \beta)\, p(\boldsymbol{\theta} \mid \alpha)\, d\boldsymbol{\theta}$$

The evidence contains the information needed to resolve uncertainty in forecasting ash clouds


Ash Cloud 1500h on May 6, 2010
Eyjafjallajokull volcano
Projected ash from different
source elevations at
00h on May 6th, 2010
Satellite
Cloud Load
kg/km2
4.2
3.8
3.4
3
2.6
2.2
1.8
1.4
1
0.6
0.2
Z
Y
X
-35
-25
-15
-5
-35
-25
-15
-5

Posterior distributions of model input data



Posterior distributions for model parameters constrained by satellite data typically have one dominant peak during any 3 hour period, with peaks separated in time

A Laplace approximation for each posterior peak (Denlinger et al., 2012) allows rapid evaluation of the evidence integral.

The second derivative $\mathbf{A}$ of the posterior is scaled by the errors and can be written

$$\mathbf{A} = -\nabla\nabla \ln p(\boldsymbol{\theta}_{MAP} \mid D(t^n), \alpha, \beta) = \alpha\mathbf{I} + \beta\mathbf{H}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{H}$ is the Hessian of the misfit

These errors determine how well satellite data constrain a forecast

For linear transport models, with *M* model parameters, *N* satellite observations, and *N* >> *M*, the optimal errors may be found by taking the derivative of the Hessian of the maximum posterior solution with respect to α or β and finding the maximum value.

For N>>M, the Hessian is a maximum at these average variances

$$\frac{1}{\alpha} = \frac{\mathbf{sm}^{\mathrm{T}}\mathbf{sm} + \mathbf{sh}^{\mathrm{T}}\mathbf{sh}}{M}$$

$$\frac{1}{\beta} = \frac{\sum_{N} \left(M(\boldsymbol{\theta}, t) - \mathbf{D}(t)\right)^2}{N}$$

This process is easily automated once satellite data are obtained

Bad error estimates degrade the forecast *F*, since

$$F(x_{ij} \mid t^m, \alpha, \beta) = \int M_{ij}(\boldsymbol{\theta}, t^m, \alpha, \beta)\ \overline{p}(\boldsymbol{\theta} \mid D(t^n), \alpha, \beta)\, d\boldsymbol{\theta}$$

contains these errors, in which forecast *F* gives an estimate that ash will be found at location x_{ij} at some future time, where $t^m > t^n$

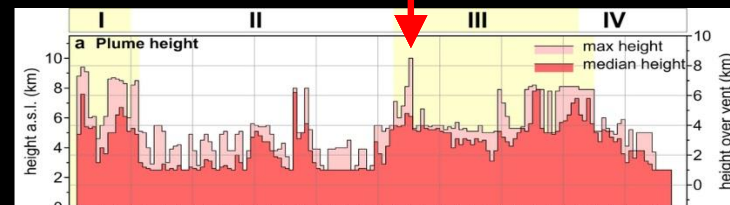
The optimal error provides the maximum data constraints and ensures that the maximum constraints available from the satellite data are used in forecasting future ash clouds

Example: How do errors affect forecasts of ash clouds produced by Eyjafjallajokull volcano from May 5-7th 2010?

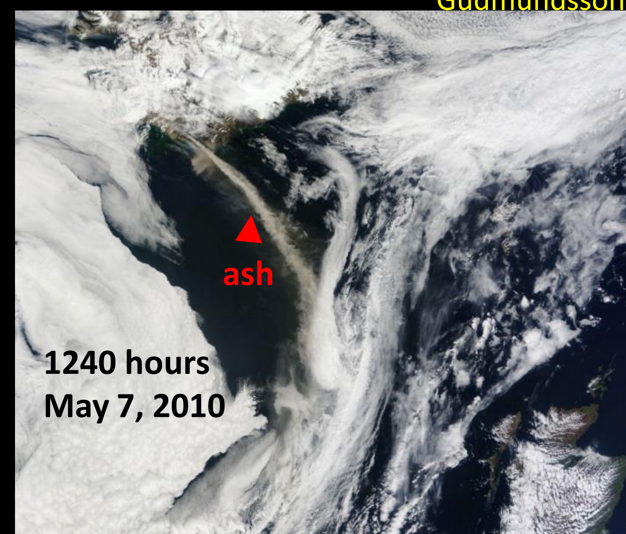
We will train the model at 1155 hours, May 6

Find α and β then make forecasts with these errors and ones that are much worse

00h on May 6, 2010

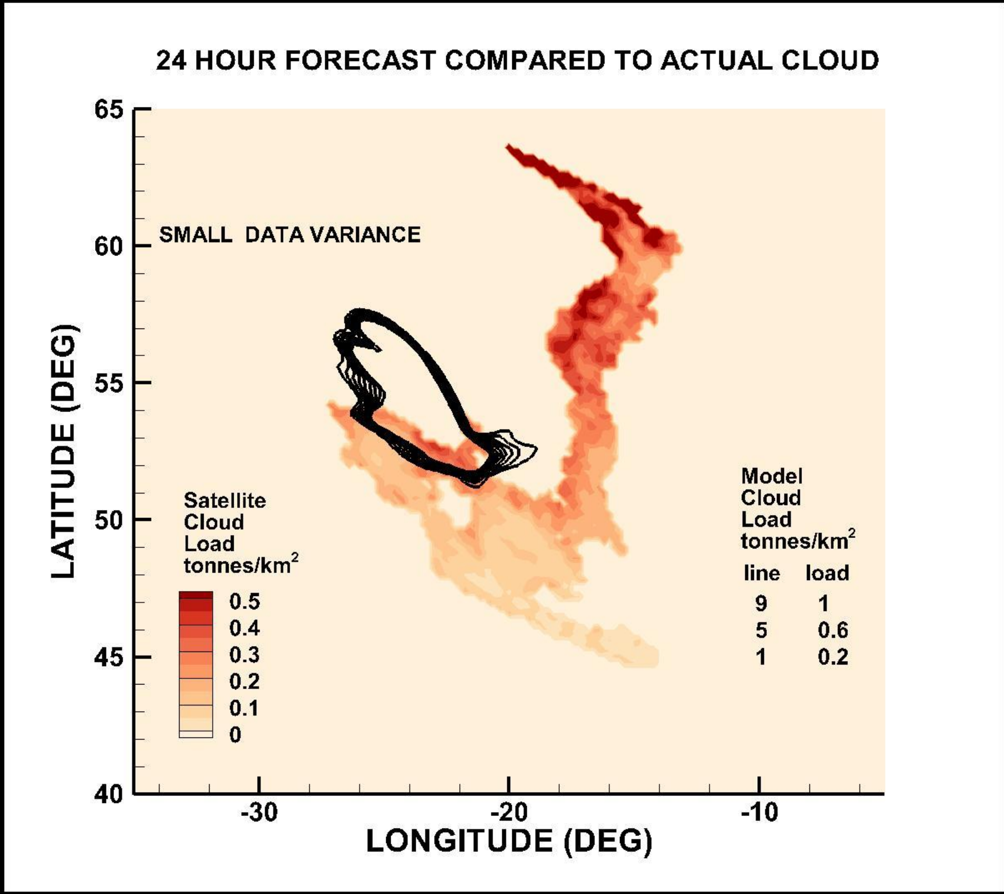


Gudmundsson et al., 2012



Result:
Underestimate the errors in the satellite, wind data, and in model transport, and the forecast is over constrained.

Potential data constraints on forecasts are lost.

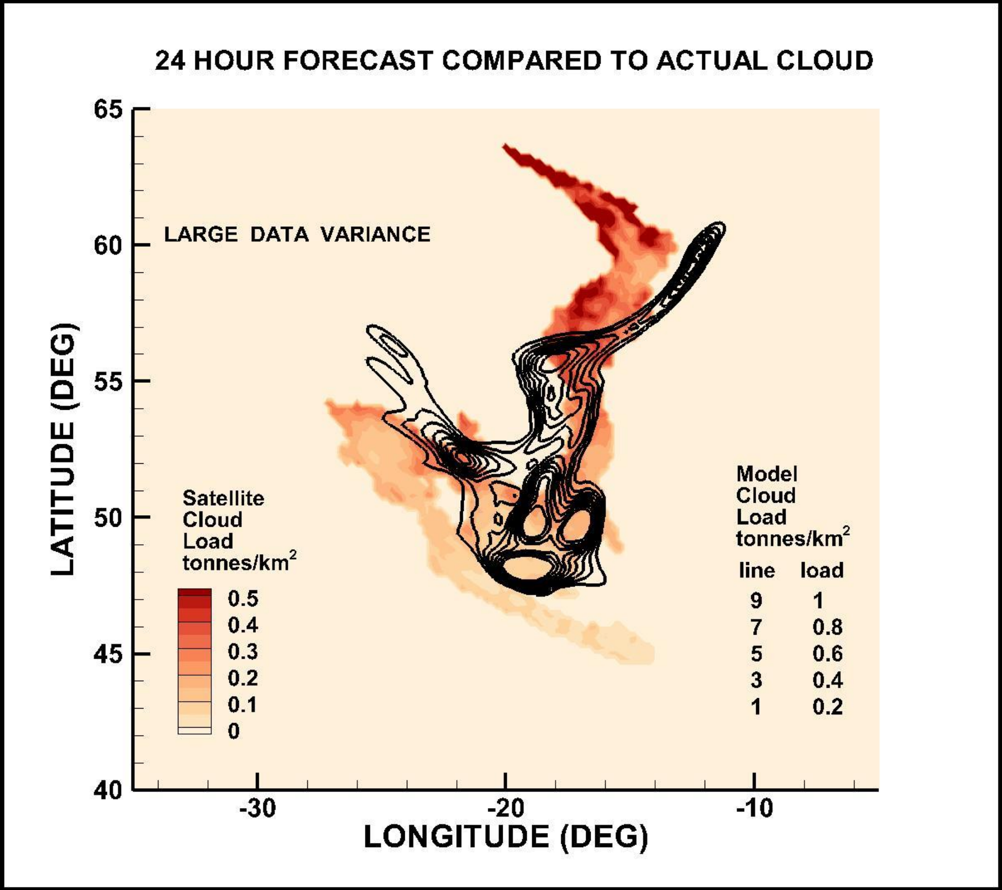


The forecast is given as black contours, whereas actual cloud is rust colored.

Result:

Overestimate the errors in the satellite, wind data, and in model transport, and the forecast is under constrained.

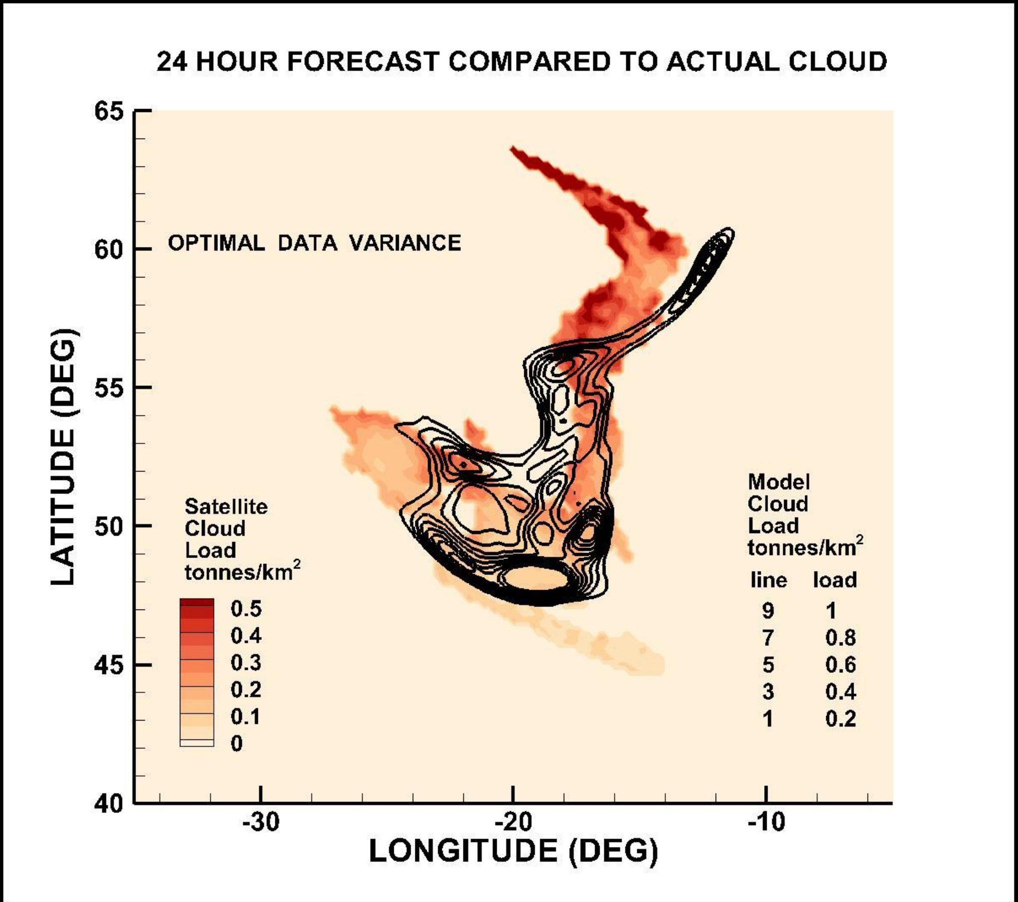
Data constraints on forecasts are weakened by too large an error.



The forecast is given as black contours, whereas actual cloud is rust colored.

Result:
The errors in the satellite, wind data, and in model transport, are appropriate for the measurements and environmental conditions.

The forecast accurately reflects the constraints of the satellite data.



The forecast is given as black contours, whereas actual cloud is rust colored.

Conclusions

Ash clouds travel more than 50 km from the volcano in a few hours or less, and in doing so lose particles > 25 microns. Forecasts 24 h or more are for clouds of fine ash particles whose settling velocity is negligible to winds.

1. **The error inherited from these satellite data directly affect forecasts**

2. **The uncertainty in forecasting is a combination of these inherited errors, errors in transport models, and errors in wind**

3. **A Bayesian framework for making forecasts also provides the means to measure misfits of models with data, and use this misfit to determine the optimal data constraints and the optimal forecast.**