# Short-Term Scientific Mission
# Scientific Report

COST Action: IS1304

STSM Title: Dependency Modelling

Reference : ECOST-STSM-IS1304-070316-070714

STSM Dates: From 07-03-2016 to 06-04-2016

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# 1 Introduction

The Short-Term Scientific Missions (STSMs) contribute to the scientific objectives of a COST Action by enabling researchers to carry out international research visits. After the research visit, the beneficiary of the STSM must submit a scientific report to the Host and to the STSM Coordinator within 30 days after the end of his/her stay.

Under the COST Action IS1304, *Expert Judgment Network: Bridging the Gap Between Scientific Uncertainty and Evidence-Based Decision Making*, the author of this report visited TU-Delft (The Netherlands), hosted by Dr. Morales Nápoles, from 7/03/2016 to 6/04/2016 (31 days).

This report aims to present the purpose of the STSM (Section 2), the work carried out (Section 3), the description of the main results obtained (Section 4), the future collaboration with the Host Institution (Section 5) and a description of the foreseen publications (Section 6). Moreover, the document contains two Appendices showing partial results.

# 2 Context and purpose of the STSM

There is a lack of consensus in relation to the operational definition of concepts and descriptors of traffic networks regarding their resilience, vulnerability and criticality. With the aim of determining a mathematically sound framework to objectively define and delimit these concepts, the structured expert judgment is proposed to assess the vulnerability of a traffic network when non disruptive events have been previously identified. Moreover, the expert judgment for dependence modelling is used to establish to what extent common indicators of the traffic network performance, such as accessibility and reliability, explain the vulnerability.

Therefore, based on the structured expert judgement elicitation, the following relevant questions are expected to be answered;

1. Is it possible to identify vulnerability of traffic networks separately from the concept of hazard?
2. Can accessibility or reliability be considered as a unifying framework for understanding and interpreting the concept of traffic network vulnerability?
3. Are accessibility and reliability enough to assess the vulnerability or is there any significant missing aspect to be considered?

It is highlighted that, though applied to the case of the metric *vulnerability*, the author expects to develop a similar methodology suitable to many other traffic indicators such as resilience, robustness, effectiveness, serviceability, etc.

# 3 Description of the work carried out during the STSM

During the STSM the following tasks were conducted;

- To review the relevant papers in the area of structured expert judgement uncertainty quantification; to analyse the questionnaires used in previous expert judgement elicitation processes; and to learn how to use the Excalibur software package.
- To overview the work done in TU-Delft regarding expert judgement elicitation of dependency modelling; and to analyse the questionnaires used in previous processes of expert judgement elicitation of dependence modelling.
- To identify the most suitable traffic indices related to the road network vulnerability, that is, *accessibility* and *reliability*; to define them removing the possible ambiguity; and to obtain mathematical formulations for their assessment.
- To develop a case study based on the national road network of Ireland to show the applicability of the methodology.
- To develop a tailor-made Matlab code for the obtention of the metrics accessibility and reliability and its statistical relations, to be used in the current application of interest.
- To draft a questionnaire to determine the uncertainty of the experts regarding the metric vulnerability of a traffic network, and the statistical dependence between the metrics vulnerability, accessibility and reliability.
- To conduct a dry run exercise with an expert on resilience of traffic networks (Researcher of TNO, The Netherlands), in order to learn how to carry out an expert elicitation. Moreover, it allowed the improvement of the questionnaire.
- To draft the outline of a paper presenting the structured expert judgement uncertainty quantification of variables and of dependency modelling as a novel methodology to reach an agreement regarding the operationality of concepts and descriptors of traffic networks.

# 4 Description of the main results obtained

With the aim of addressing the questions raised in Section 2, the case study selected was the national road network of Ireland, during the interval of time 8.00 a.m. to 9.00 am (working days). The required information, such as the network geometry and road characteristics, were obtained from AECOM and ESRI (2014). The traffic data correspond to January 2016 according to the NRA traffic database. A detailed description of the case study can be found in Appendix 2.

Using the Monte Carlo method, 10000 simulations were carried out. The combination of different traffic demands were introduced to obtain the travel time and the link flow associated with the links and routes of the traffic network. For each simulation, the indices reliability and accessibility associated with the set of origin-destination (OD) pairs were computed. The data obtained, which are shown in Appendix 1, were used for the preparation of the elicitation process.

The elicitation consists of two parts, (a) elicitation of uncertainty, where the Cook method will be applied to determine the vulnerability associated with different OD pairs, and (b) elicitation of dependence modelling, where the statistical relations between vulnerability, accessibility and reliability will be studied. In this case, the dependence relations between variables are modelled directly by Gaussian copulas, and the score of the experts will be computed considering the D-Calibration score (see Morales-Nápoles et al. (2013)).

Therefore, the questionnaire (see Appendix 2) consists on 10 questions for calibration of uncertainty, 6 questions for calibration of dependence, 5 questions related to the variables of interest and finally, 10 questions on dependence between variables. Table 1 summarizes the structure of the questionnaire. $A_{ij}$, $R_{ij}$ and $V_{ij}$ denote accessibility, reliability and vulnerability associated with the OD pair $ij$.

Table 1: Structure of the questionnaire. $A_{ij}$, $R_{ij}$ and $V_{ij}$ denote accessibility, reliability and vulnerability associated with the OD pair $ij$.

Elicitation of Uncertainty

| | Calibration Variables | | Variables of Interest |
|---|---|---|---|
| ODs | $Max[A_{ij}]$ | $Max[R_{ij}]$ | $V_{ij}$ (percentiles 5, 50 and 95) |
| 20-25 | 0.590 | 1.125 | |
| 25-69 | 0.457 | 1.183 | |
| 32-69 | 0.816 | - | *Unknown values* |
| 32-92 | 0.636 | 1.211 | |
| 69-92 | 0.851 | 1.287 | |
| All | - | 1.325$^{(*)}$ | - |

Elicitation of Dependence Modelling

| Calibration Variables | | Variables of Interest (percentile 50) | | |
|---|---|---|---|---|
| | | ODs | $Prob(V_{i,j}\|A_{i,j})$ | $Prob(V_{i,j}\|A_{i,j}, R_{i,j})$ |
| $Prob(A_{25,69}\|A_{32,92})$ | 0.499 | 20-25 | | |
| $Prob(A_{32,92}\|A_{69,92})$ | 0.455 | 25-69 | | |
| $Prob(A_{25,69}\|A_{32,92}, A_{69,92})$ | 0.500 | 32-69 | *Unknown values* | |
| $Prob(R_{25,69}\|R_{32,92})$ | 0.575 | 32-92 | | |
| $Prob(R_{32,92}\|R_{69,92})$ | 0.871 | 69-92 | | |
| $Prob(R_{25,69}\|R_{32,92}, R_{69,92})$ | 0.563 | | | |

(∗) The maximum reliability of the network is included to increase the variability of the elicited values.

The results presented in Appendix 1 show that the accessibility indices for the ODs analysed are very independent, whereas a higher correlation exists between the reliability indices.

After conducting the interviews, the results obtained will allow the assessment of the metric vulnerability and its relation with the metrics reliability and accessibility.

## 5 Future collaboration with TU-Delft

In the coming months, the author of this report expects to accomplish the following tasks, which will be carried out with the collaboration and guidance of the Host;

- To conduct 5 interviews with experts to elicit their judgment according to the questionnaire shown in Appendix 2.

- To analyse the results of the interviews, using the Excalibur software package and the tailor-made Matlab code developed.
- To participate and present preliminary results in the workshop on expert judgment elicitation, hosted by TU-Delft, in June, $22^{nd} - 24^{th}$, 2016.

# 6 Foreseen publications/articles resulting from the STSM

It is expected that the results obtained during the research visit will be disseminated as follows;

- Journal paper, titled *Understanding the vulnerability of traffic networks by means of structured expert judgment elicitation.*
- Conference paper in the Irish Transport Research Network Conference 2016, September, $1^{st} - 2^{nd}$, 2016 in Dublin.
- Conference paper, titled *Structured expert judgment for dependence modelling applied to the selection of indicators of infrastructure resilience.* This has been already accepted for its presentation on the Ninth International Forum on Engineering Decision Making: Resilient Infrastructures - Integration of Risk and Sustainability, December, $7^{th} - 10^{th}$, 2016 in Switzerland.

# References

AECOM and ESRI (2014). National transport model, model development report. Tech. Rep. 1, National Roads Authority, Ireland.

Morales-Nápoles, O., Hanea, A. M., and Worm, D. T. H. (2013). Experimental results about the assessments of conditional rank correlations by experts: Example with air pollution estimates. In *ESREL 2013: Proceedings of the 22nd European Safety and Reliability Conference" Safety, Reliability and Risk Analysis: Beyond the Horizon", Amsterdam, The Netherlands, 29 september-2 oktober 2013*. CRC Press/Balkema-Taylor & Francis Group.

# Appendix 1. Numerical Analysis

## I. Statistical relations between ACC/ACC, REL/REL & ACC/REL

```
Variables for calibration of the dependence modelling:{25-69, 32-92, 69-92}
25-69 depends of 32-92 and 69-92. 32-92 depends of 69-92.

--- CORRELATION ACC/ACC ------------
         [25-69] [32-92] [69-92]
 [25-69]  1.000  -0.007  -0.018
 [32-92] -0.007   1.000  -0.166
 [69-92] -0.018  -0.166   1.000

 Prob(A_{25,69}|A_{32,92})=0.4996
 Prob(A_{32,92}|A_{69,92})=0.4548
 Prob(A_{25,69}|A_{32,92},A_{69,92})=0.5004

*Dependece elicitation;
 r(A_{69,92},A_{32,92}) = -0.13526
 r(A_{32,92},A_{25,69}) = -0.00120
 P(A_{25,69}>med | A_{32,92}>med, A_{69,92}>med) is in the interval (0.01862,0.98115)
 Conditional independence at 0.49962
 r(A_{25,69},A_{69,92}) = 0.00251
 r(A_{69,92},A_{25,69} | A_{32,92}) = 0.00225
 D-Calibration: 0.9863

--- CORRELATION REL/REL ------------
         [25-69] [32-92] [69-92]
 [25-69]  1.000   0.240   0.115
 [32-92]  0.240   1.000   0.916
 [69-92]  0.115   0.916   1.000

 Prob(R_{25,69}|R_{32,92})=0.5752
 Prob(R_{32,92}|R_{69,92})=0.8712
 Prob(R_{25,69}|R_{32,92},R_{69,92})=0.5627

*Dependece elicitation;
 r(R_{69,92},R_{32,92}) = 0.91205
 r(R_{32,92},R_{25,69}) = 0.22402
 P(R_{25,69}>med | R_{32,92}>med, R_{69,92}>med) is in the interval (0.51302,0.65947)
 Conditional independence at 0.58277
 r(R_{25,69},R_{69,92}) = 0.10660
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r(R_{69,92},R_{25,69} | R_{32,92}) = -0.27165
D-Calibration: 0.9807
```

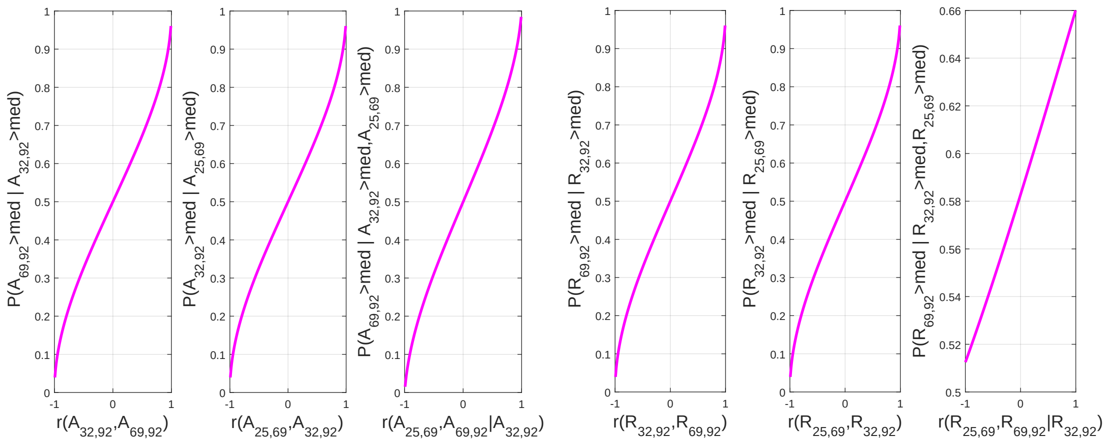


(a) Accessibility (b) Reliability

Figure 1: Statistical relations between correlation and its associated conditional probability of (a) accessibility and (b) reliability for the selected OD's.

## II. Correlation ACC/REL & Seed variables ACC&REL

--- CORRELATION ACC/REL & SEED VARIABLES ACC&REL ------------

| | [20-25] | [25-69] | [32-69] | [32-92] | [69-92] |
|---|---|---|---|---|---|
| [c_A&R] | 0.968 | 0.631 | 0.680 | 0.674 | 0.703 |
| [Med_A] | 0.590 | 0.457 | 0.816 | 0.636 | 0.851 |
| [Max_A] | 0.605 | 0.477 | 0.846 | 0.668 | 0.880 |
| [Med_R] | 1.108 | 1.159 | 1.232 | 1.140 | 1.187 |
| [Max_R] | 1.125 | 1.183 | 1.272 | 1.211 | 1.287 |
| [Re\|Ac] | 0.929 | 0.728 | 0.746 | 0.757 | 0.765 |

# Appendix 2. Questionnaire

## 1 Introduction

This questionnaire is concerned with the elicitation of uncertainty distributions over statistical dependence measures between values of vulnerability, accessibility and reliability of road traffic networks. More precisely, this questionnaire focusses on the metrics associated with different OD (origin-destination) pairs of an inter-urban network.

This document is organised as follows; Section 2 provides some clarifications regarding the issue addressed; the road traffic network, the characteristics required to answer the questionnaire and the definition of the different case studies are given in Section 3; finally, the set of questions of interest for each case are presented in Section 4 onwards.

## 2 Definitions

The FUNCTION of a traffic (sub-) network is to provide accessibility for the traffic (sub-) network users with a given level of service. This level of service will be measured by means of the travel times experienced by the users.

VULNERABILITY of a (sub-) network is the susceptibility to incidents that can result in considerable reduction or loss of its functionality. The incidents considered (vehicle breakdowns, crashes, roadworks, severe weather, terrorist attacks, etc.) are characterised by their random occurrence in space and time. Note that big disasters might affect the accessibility to basic services, meanwhile light perturbations result in a decrease of the reliability of the traffic systems.

Given an incident in a random location of the network (not necessary between the OD pair $ij$), an OD pair $ij$ with a null vulnerability ($V_{ij} = 0$) implies either that no user driving from $i$ to $j$ is affected by the incident, or the level of service experienced by users driving from $i$ to $j$ is not reduced as a consequence of the incident. An OD pair $ij$ is completely vulnerable ($V_{ij} = 1$) when the OD pair $ij$ loses completely its functionality as a consequence of the incident.

ACCESSIBILITY of a (sub-) network is the ease for road users to reach certain services from specific locations (origins) by using the traffic (sub-) network at a specific time. The services considered are (a) business, (b) education, (c) health services and (d) interconnection with other modes. A null accessibility from the origin i to the destination $j$ ($A_{ij} = 0$) implies that users cannot reach any of the services considered when travelling from $i$ to $j$. A total accessibility from the origin i to the destination $j$ ($A_{ij} = 1$) implies that the set of services are all located in the origin $i$, and all users of the network are located in that origin.

(Travel time) RELIABILITY measures the feasibility that road users reach a destination within a certain travel time under the operating conditions encountered. To measure the reliability of a given OD pair $ij$, the actual travel time experienced by users travelling from origin $i$ to the destination $j$, $t_{ij}$, is compared with the associated travel time in free flow conditions, $ft_{ij}$, that is, $t_{ij}/ft_{ij}$. The minimum value of the reliability associated with the OD pair $ij$ is $R_{ij} = 1$, reached when the level of service is A according to the North American Highway Capacity Manual. Note that the reliability is not upper bounded.

## 3 Definition of the case studies

The traffic network presented in Figure 2 is under study during the interval of time 8.00 a.m. to 9.00am (working days). The lengths of the links are proportional to the real length of the roads connecting the corresponding nodes, and all links represent bidirectional roads. Besides, the following information is given, (a) the characteristics of the links (roads) for good ambient conditions (see Figure 3), (b) the OD pairs and the probabilistic distribution of demands associated to each OD pair in the interval of time studied (see Figure 4). Additional notes; (1) a static traffic assignment model is used to reproduce the traffic during the period of interest, based on a deterministic User Equilibrium approach. (2) It is assumed that, during the time interval analysed, the 60% of the demand is travelling because of business reasons, and 12% because of educational purposes, and 3% and 8% of the demand are the potential users of the health services and inter-modality facilities, respectively. (3) The accessibility of the OD pair $ij$ at the time interval studied is calculated according the following expression (see Figure 5).

$$A_{ij} = sum_s \left[ \left( \frac{D_{ik}}{D} \right)^b \exp(-\alpha t_{ik}) \right], \tag{1}$$

where $D_{ik}$ is the demand associated with the service $s$, departing from node $i$ and reaching the closest service in node $k$ when travelling from $i$ to $j$. $D$ is the total demand of the network, $t_{ik}$ is the time required to cover the distance between nodes $i$ and $k$, and $b$ and $\alpha$ are scale factors. In this case study $b = 0.2$ and $\alpha = 1$ hours$^{(-1)}$.

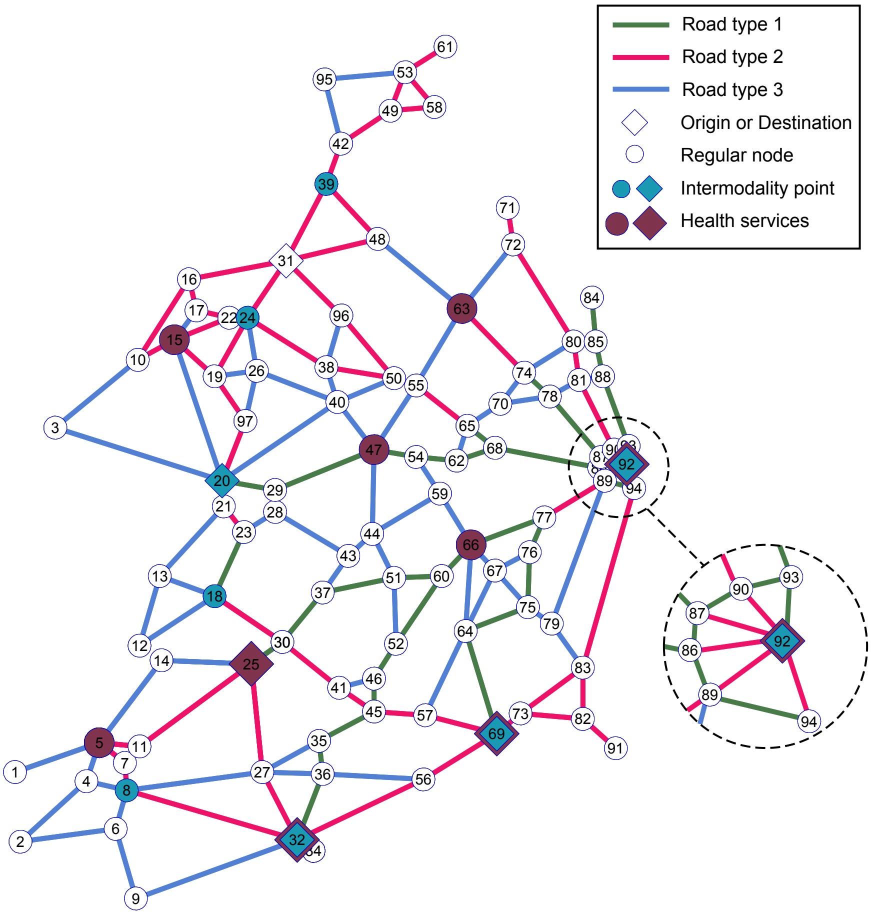


Figure 2: Traffic network under study

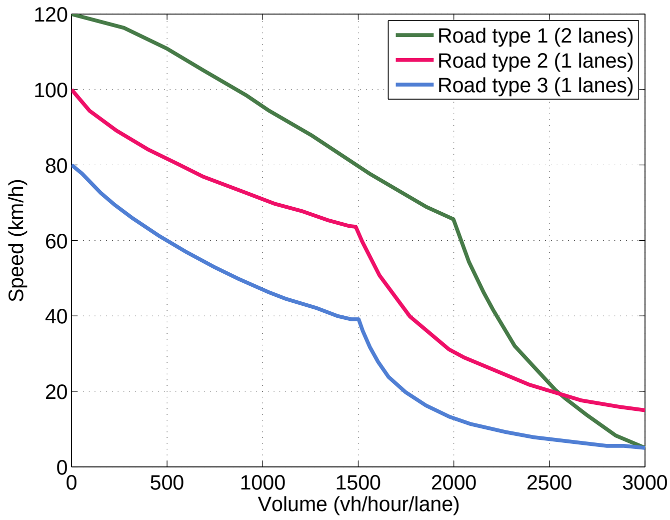


Figure 3: Characteristics of the road types

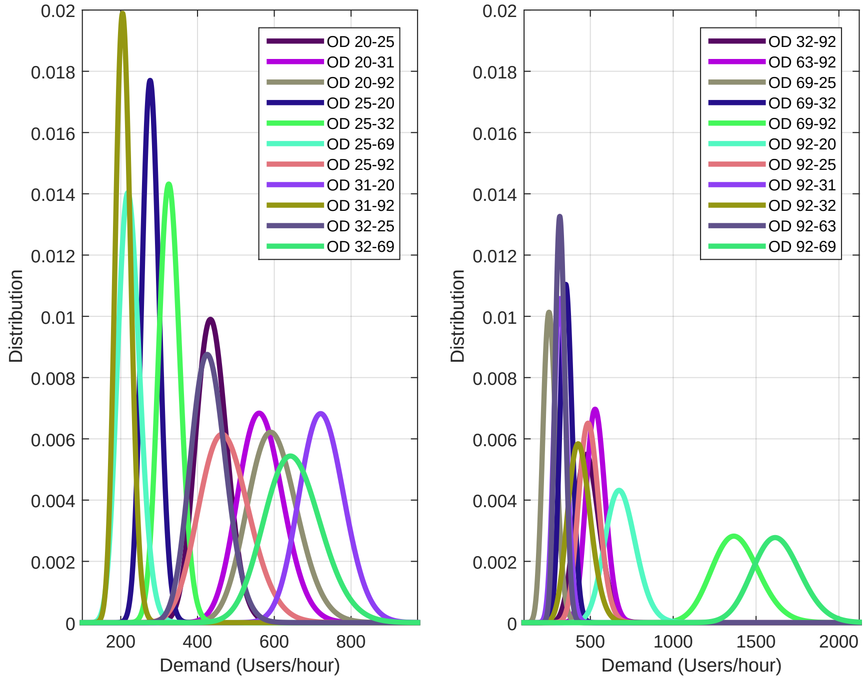


Figure 4: Distribution of demands associated with the OD pairs

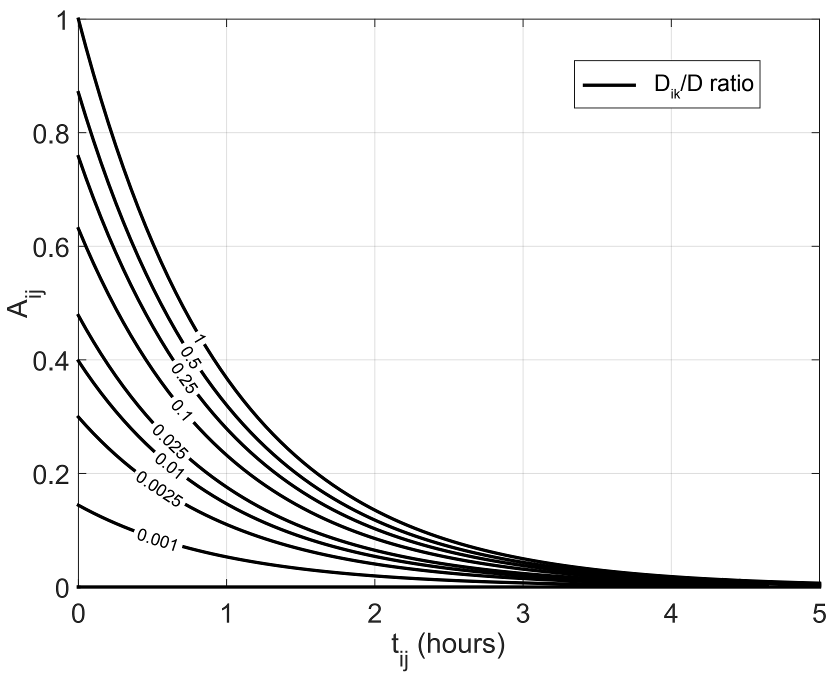


Figure 5: Contribution of a given service on the accessibility index as a function of the time, for different $\frac{D_{ik}}{D}$ ratios (contour curves).

# 4 Case study 1; OD 20-25

According to the definitions provided, for the OD pair 20-25,
*(VARIABLES FOR CALIBRATION)*

a) **(Uncertainty distribution of max$[\mathbf{A_{20,25}}]$)** What is the maximum value of the accessibility associated with the OD 20-25, $max[A_{20,25}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

b) **(Uncertainty distribution of max$[\mathbf{R_{20,25}}]$)** What is the maximum value of the reliability associated with the OD 20-25, $max[R_{20,25}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

*(VARIABLES AND DEPENDENCIES BETWEEN VARIABLES OF INTEREST)*

c) **(Uncertainty distribution of $\mathbf{V_{20,25}}$)** What is the vulnerability associated with the OD 20-25, $V_{20,25}$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

d) $\mathbf{Pr(V_{20,25} > med|A_{20,25} > 0.590)}$ What is your estimate that the vulnerability associated with OD 20-25 is larger than your estimation of the median of $V_{20,25}$ given that the accessibility associated with OD 20-25 is larger than 0.590?

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e) $\mathbf{Pr(V_{20,25} > med|A_{20,25} > 0.590, R_{20,25} > 1.125)}$ What is your estimate that the vulnerability associated with OD 20-25 is larger than your estimation of the median of $V_{20,25}$ given that both, the accessibility and the reliability associated with OD 20-25 are larger than 0.590 and 1.125, respectively?

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# 5 Case study 2; OD 25-69

According to the definitions provided, for the OD pair 25-69,
*(VARIABLES FOR CALIBRATION)*

a) **(Uncertainty distribution of $\mathbf{max[A_{25,69}]}$)** What is the maximum value of the accessibility associated with the OD 25-69, $max[A_{25,69}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

b) **(Uncertainty distribution of $\mathbf{max[R_{25,69}]}$)** What is the maximum value of the reliability associated with the OD 25-69, $max[R_{25,69}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

*(VARIABLES AND DEPENDENCIES BETWEEN VARIABLES OF INTEREST)*

c) **(Uncertainty distribution of $\mathbf{V_{25,69}}$)** What is the vulnerability associated with the OD 25-69, $V_{25,69}$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

d) $\mathbf{Pr(V_{25,69} > med|A_{25,69} > 0.457)}$ What is your estimate that the vulnerability associated with OD 25-69 is larger than your estimation of the median of $V_{25,69}$ given that the accessibility associated with OD 25-69 is larger than 0.457?

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e) $\mathbf{Pr(V_{25,69} > med|A_{25,69} > 0.457, R_{25,69} > 1.183)}$ What is your estimate that the vulnerability associated with OD 25-69 is larger than your estimation of the median of $V_{25,69}$ given that both, the accessibility and the reliability associated with OD 25-69 are larger than 0.457 and 1.183, respectively?

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## 6 Case study 3; OD 32-69

According to the definitions provided, for the OD pair 32-69,
*(VARIABLES FOR CALIBRATION)*

a) **(Uncertainty distribution of $\mathbf{max[A_{32,69}]}$)** What is the maximum value of the accessibility associated with the OD 32-69, $max[A_{32,69}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

*(VARIABLES AND DEPENDENCIES BETWEEN VARIABLES OF INTEREST)*

b) **(Uncertainty distribution of $\mathbf{V_{32,69}}$)** What is the vulnerability associated with the OD 32-69, $V_{32,69}$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

c) $\mathbf{Pr(V_{32,69} > med|A_{32,69} > 0.816)}$ What is your estimate that the vulnerability associated with OD 32-69 is larger than your estimation of the median of $V_{32,69}$ given that the accessibility associated with OD 32-69 is larger than 0.816?

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d) $\mathbf{Pr(V_{32,69} > med|A_{32,69} > 0.816, R_{32,69} > 1.272)}$ What is your estimate that the vulnerability associated with OD 32-69 is larger than your estimation of the median of $V_{32,69}$ given that both, the accessibility and the reliability associated with OD 32-69 are larger than 0.816 and 1.272, respectively?

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# 7 Case study 4; OD 32-92

According to the definitions provided, for the OD pair 32-92,
*(VARIABLES FOR CALIBRATION)*

a) **(Uncertainty distribution of max[$\mathbf{A_{32,92}}$])** What is the maximum value of the accessibility associated with the OD 32-92, $max[A_{32,92}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

b) **(Uncertainty distribution of max[$\mathbf{R_{32,92}}$])** What is the maximum value of the reliability associated with the OD 32-92, $max[R_{32,92}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

*(VARIABLES AND DEPENDENCIES BETWEEN VARIABLES OF INTEREST)*

c) **(Uncertainty distribution of $\mathbf{V_{32,92}}$)** What is the vulnerability associated with the OD 32-92, $V_{32,92}$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

d) $\mathbf{Pr(V_{32,92} > med|A_{32,92} > 0.636)}$ What is your estimate that the vulnerability associated with OD 32-92 is larger than your estimation of the median of $V_{32,92}$ given that the accessibility associated with OD 32-92 is larger than 0.636?

e) $\mathbf{Pr(V_{32,92} > med|A_{32,92} > 0.636, R_{32,92} > 1.211)}$ What is your estimate that the vulnerability associated with OD 32-92 is larger than your estimation of the median of $V_{32,92}$ given that both, the accessibility and the reliability associated with OD 32-92 are larger than 0.636 and 1.211, respectively?

# 8 Case study 5; OD 69-92

According to the definitions provided, for the OD pair 69-92,
*(VARIABLES FOR CALIBRATION)*

a) **(Uncertainty distribution of max[$\mathbf{A_{69,92}}$])** What is the maximum value of the accessibility associated with the OD 69-92, $max[A_{69,92}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

b) **(Uncertainty distribution of max[$\mathbf{R_{69,92}}$])** What is the maximum value of the reliability associated with the OD 69-92, $max[R_{69,92}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

*(VARIABLES AND DEPENDENCIES BETWEEN VARIABLES OF INTEREST)*

c) **(Uncertainty distribution of $\mathbf{V_{69,92}}$)** What is the vulnerability associated with the OD 69-92, $V_{69,92}$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

d) $\mathbf{Pr(V_{69,92} > med|A_{69,92} > 0.851)}$ What is your estimate that the vulnerability associated with OD 69-92 is larger than your estimation of the median of $V_{69,92}$ given that the accessibility associated with OD 69-92 is larger than 0.851?

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e) $\mathbf{Pr(V_{69,92} > med|A_{69,92} > 0.851, R_{69,92} > 1.287)}$ What is your estimate that the vulnerability associated with OD 69-92 is larger than your estimation of the median of $V_{69,92}$ given that both, the accessibility and the reliability associated with OD 69-92 are larger than 0.851 and 1.287, respectively?

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# 9 Combined cases

*(VARIABLES FOR CALIBRATION)*

a) **(Uncertainty distribution of** $\mathbf{max[R_{i,j}]}$**)** What is the maximum value of the reliability associated with ALL ODs of the network, $max[R_{i,j}]$? Provide the 5th, 50th and 95th percentiles of the uncertainty distribution.

| $5^{th}$ | $50^{th}$ | $95^{th}$ |
|---|---|---|
| | | |

*(DEPENDENCIES BETWEEN VARIABLES FOR CALIBRATION)*

a) $\mathbf{Pr(A_{25,69} > 0.457 | A_{32,92} > 0.636)}$ What is your estimate that the accessibility associated with OD 25-69 is larger than 0.457 given that the accessibility associated with OD 32-92 is larger than 0.636?

b) $\mathbf{Pr(A_{32,92} > 0.636 | A_{69,92} > 0.851)}$ What is your estimate that the accessibility associated with OD 32-92 is larger than 0.636 given that the expected value of the accessibility associated with OD 69-92 is larger than 0.851?

c) $\mathbf{Pr(A_{25,69} > 0.457 | A_{69,92} > 0.851, A_{32,92} > 0.636)}$ What is your estimate that the accessibility associated with OD 25-69 is larger than 0.457 given that (a) the accessibility associated with OD 32-92 is larger than 0.636, and (b) the expected value of the accessibility associated with OD 69-92 is larger than 0.851?

d) $\mathbf{Pr(R_{25,69} > 1.183 | R_{32,92} > 1.211)}$ What is your estimate that the reliability associated with OD 25-69 is larger than 1.183 given that the reliability associated with OD 32-92 is larger than 1.211?

e) $\mathbf{Pr(R_{32,92} > 1.211 | R_{69,92} > 1.287)}$ What is your estimate that the reliability associated with OD 32-92 is larger than 1.211 given that the expected value of the reliability associated with OD 69-92 is larger than 1.287?

f) $\mathbf{Pr(R_{25,69} > 1.183|R_{69,92} > 1.287, R_{32,92} > 1.211)}$ What is your estimate that the reliability associated with OD 25-69 is larger than 1.183 given that (a) the reliability associated with OD 32-92 is larger than 1.211, and (b) the expected value of the reliability associated with OD 69-92 is larger than 1.287?