On combining elicitation of judgment and robustness analysis

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Summary

- The nature of decision aiding
- Elicitation difficulties
- Robustness analysis concepts
- The role of robustness analysis in decision aiding
- Illustration for an additive aggregation model
- Conclusions

Overall purpose of decision aiding

To have a structured process to gather information and think about a decision

To gain insights about the decision

Language for communicating and reasoning

Grounds for justifying a decision

Rather than:

Tell a decision maker what to choose

Discover an objectively optimal solution

Types of problem

Choice / Selection: select best

- selecting a project,
- choosing a location

Ranking: rank from best to worst

- a prioritization of projects (from highest priority to least),
- a ranking of universities,…

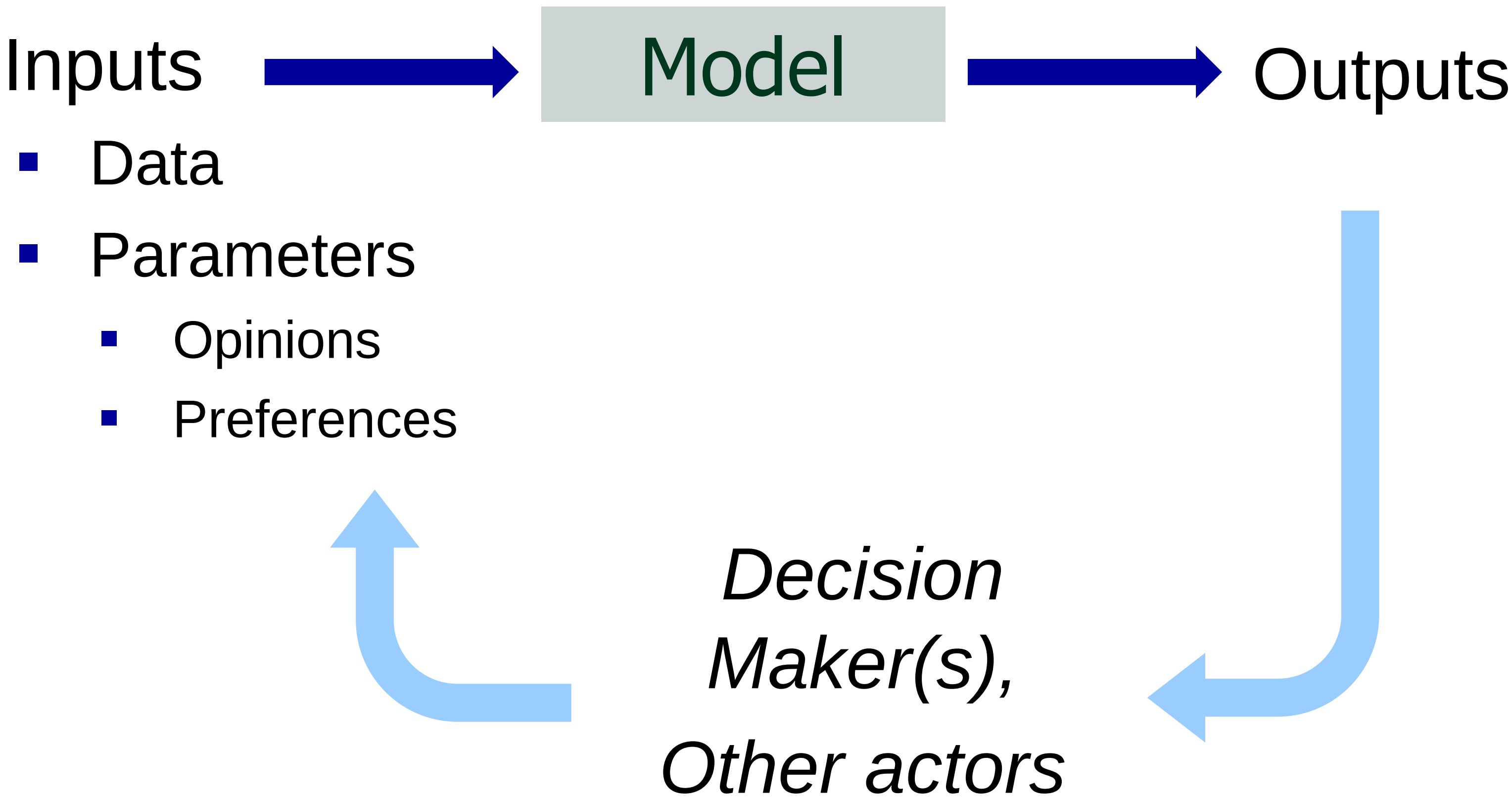
Classification / Sorting: assign to categories

- a prioritization projects *Low*, *High*, *Very High* priority class,
- land suitability maps,
- environmental rating / labelling,…

Decision aiding toolbox

- Simulation
- Optimization
  - Including Single-objective Decision Analysis
- Multi-criteria Decision Analysis
- Other more specific methods methods
  - Cost-Benefit Analysis,
  - Life Cycle Assessment, etc

Decision aiding method



Setting the model's parameters

Parameter values define

- The importance of each criterion (e.g., weights),
- Consequences (incl. probability distributions),
- External references (e.g. targets),
- Time horizon,
- Discount rate, …

Many such parameters reflect values and opinions

Setting the model’s parameters

Decision maker’s judgment (elicited)

Stakeholders and/or experts judgment (elicited)

- Polled using questionnaires
- Elicited in workshops

Society’s judgment (inferred from:)

- Market prices
- Purchase decisions (revealed preferences)
- Surveys (stated preferences)
- Political willingness to pay (e.g. taxes)
- …

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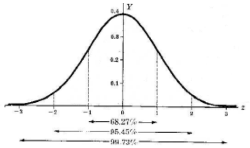
Elicitation difficulties: technical parameters, data

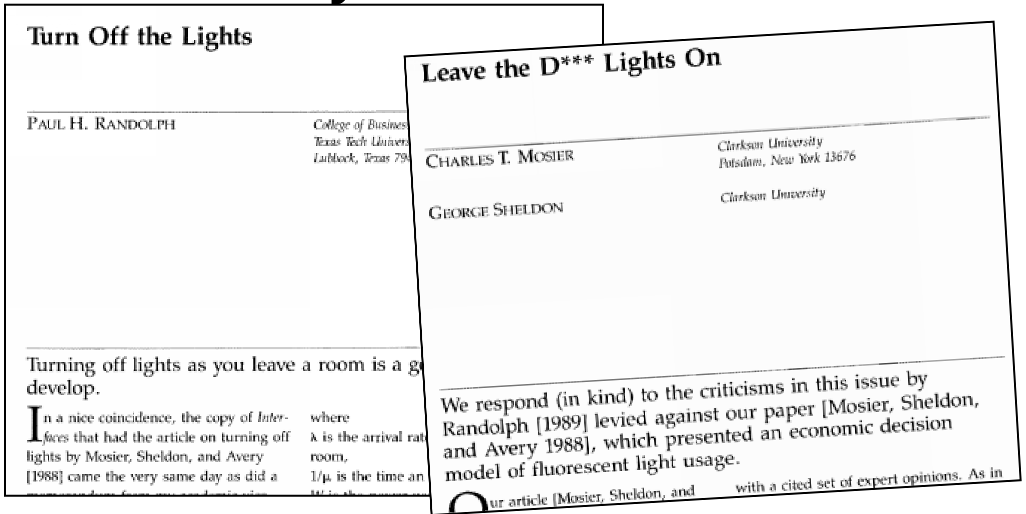
Imprecision (instruments and statistics)

Variability



Unknown future

Controversial or contradictory information



Turn Off the Lights

PAUL H. RANDOLPH — College of Busines… Texas Tech Univers… Lubbock, Texas 79…

Turning off lights as you leave a room is a g… develop.

In a nice coincidence, the copy of *Interfaces* that had the article on turning off lights by Mosier, Sheldon, and Avery [1988] came the very same day as did a …

where λ is the arrival rat… room, 1/μ is the time an…

Leave the D*** Lights On

CHARLES T. MOSIER — Clarkson University, Potsdam, New York 13676

GEORGE SHELDON — Clarkson University

We respond (in kind) to the criticisms in this issue by Randolph [1989] levied against our paper [Mosier, Sheldon, and Avery 1988], which presented an economic decision model of fluorescent light usage.

Our article [Mosier, Sheldon, and … with a cited set of expert opinions. As in

Elicitation difficulties: technical parameters, data

There is often subjectivity

e.g., measuring noise:

- At what distance?
- At what time (maximum?, average?)
- dB, dB(A) or sone?
- “Noise is the noise of others and one's dog makes no noise”

Elicitation difficulties: preferences (and also beliefs)

Framing issues, biases

Correct interpretation of parameter meaning (e.g., discount rate, scaling constant, …)

Imprecision of natural language (e.g., likely, probable, …) and poor fluency

Poor numeracy

Elicitation difficulties: preferences (and also beliefs)

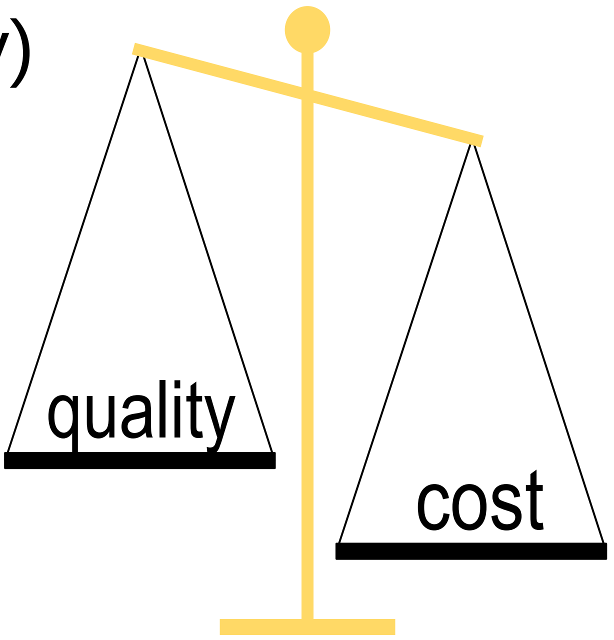
Reluctance to divulging precise parameter values in public

Lack of time availability

Lack of patience

Elicitation difficulties:
preferences (and also beliefs)

Criteria weighting often depend on concerns about the future (uncertainty)



Beliefs also may depend on what you wish

Elicitation difficulties: group decisions

Diversity of preferences

Different perceptions of reality

Hidden agendas, competition

Group phenomena and biases (inhibition, groupthink, …)

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Multiple model versions

Parameter values can vary:

- Discrete set of scenarios
- Continuous subset of parameter space
  - Ordinal information (e.g., a > b)
  - Other types of constraints
  - Results to be reproduced

Robustness analysis: different perspectives

Finding solutions “which are robust in a quite large variety of circumstances” (Beer 1966)

Flexibility in sequential decisions (Rosenhead 1988)

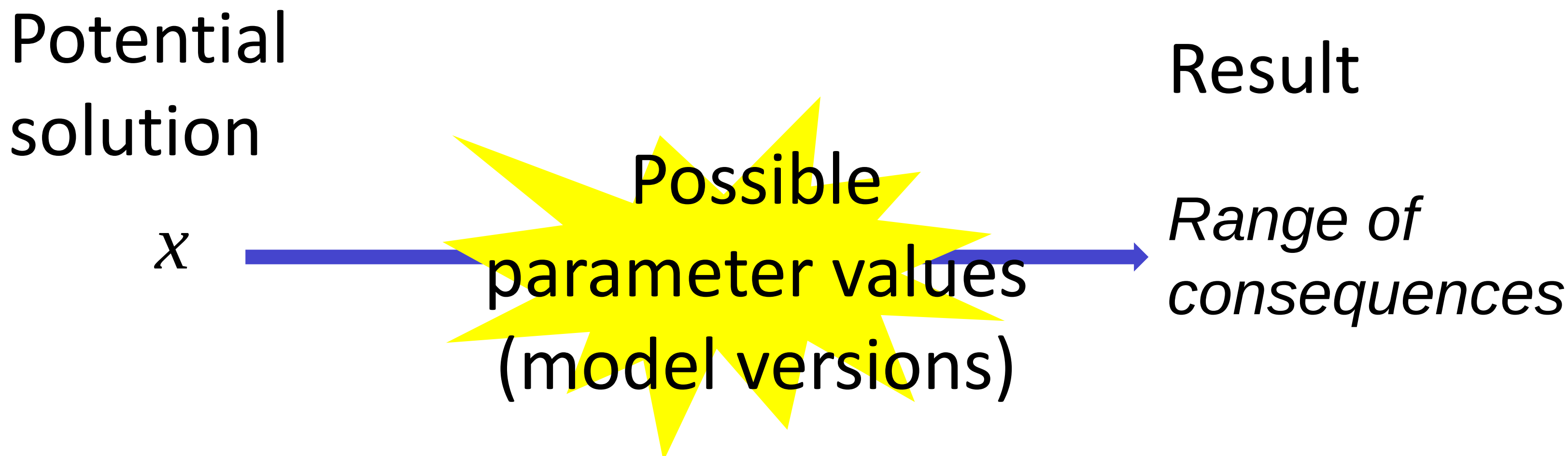
Compromise between feasibility and value of solution in optimization (Mulvey et al. 1995)

Ensuring optimal worst-case performance in optimization problems (Kouvelis & Yu 1997)

Robust vs. fragile conclusions of an analysis (Roy, 1998)

Robustness in Bayesian analyses (French, Rios Insua, Ruggeri, 2000s)

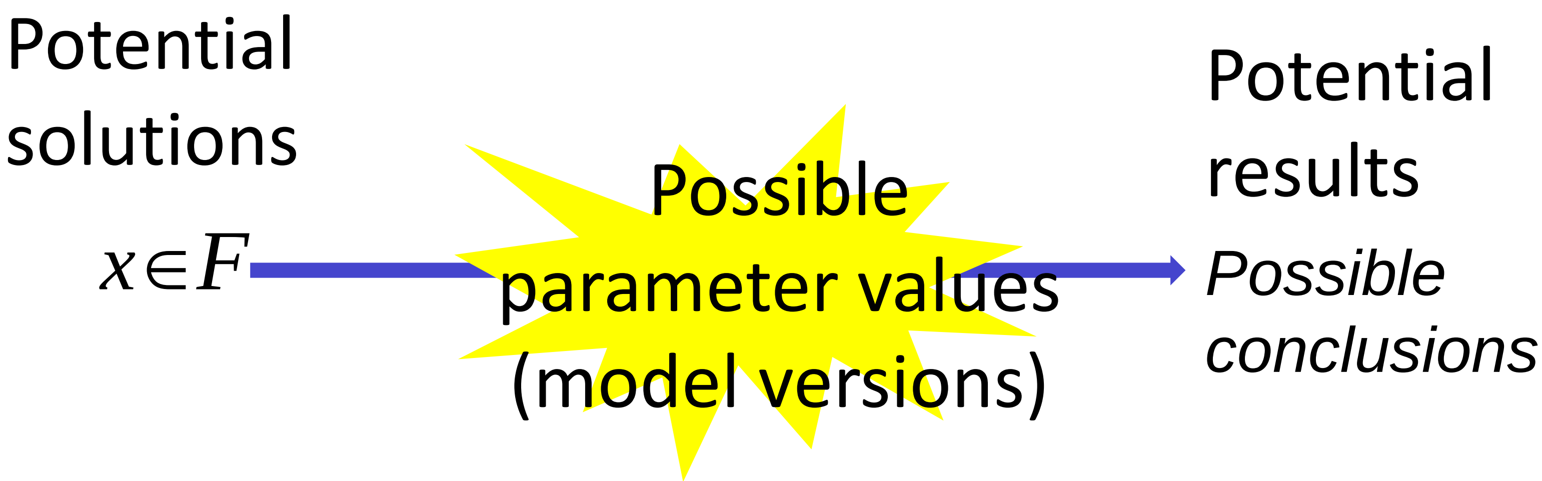
Robustness analysis
The perspective of determining the robust **solution**



Optimization process to find the best solution $x \in F$ according to a robustness criterion considering possible parameter vectors $s \in S$,

e.g. maximize minimum value $x_a = \arg \max_{x \in \bigcap_{\{s \in S\}} F_s} \min_{s \in S} f(x, s)$

Robustness analysis
The perspective of determining robust **conclusions**



What conclusions hold for all $s \in S$, e.g,

$f(x_i,s) > 10$ or $f(x_i,s) > f(x_j,s)$ or $f(x_i,s) > 0.9 \max_j f(x_j,s)$

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Typical use of robustness analysis in decision aiding processes

RA as an *ex-post* activity as the reverse of SA (Roy and Bouyssou, 1993)

After obtaining a result, to check how the result changes for selected variants of the inputs.

RA imbedded in a model to be optimized (Kouvelis & Yu 1997; Mulvey et al. 1995)

(Before obtaining a result) a model is built in order to provide, by design, a solution that is robust, e.g., the solution maximizing minimum value.

A different role for robustness analysis in decision aiding

RA as a tool to guide a decision process

- To start with little information (most reliable), postponing difficult elicitation questions (allowing to learn before answering)
- Showing the different sensitivity of conclusions and what is robust
- Motivating elicitation questions
- Progressively narrowing the range of acceptable values for the parameters

… and group decision aiding processes

RA as a tool to guide a group decision process

Postponing or avoiding difficult elicitation questions

- Postponing conflict-bound questions

Showing the different sensitivity of conclusions

- Showing where disagreement is stronger

Motivating elicitation questions

- Motivating issues to be discussed

Progressively narrowing the range of acceptable values for the parameters

- Progressing towards agreement

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Additive aggregation model

$$V(a_x) = k_1.v_1(a_x) + k_2.v_2(a_x) + \ldots + k_n.v_n(a_x),$$

e.g.,

Expected value of discrete probability distribution

Expected utility of a lottery

Additive multiattribute model

Additive group decision model

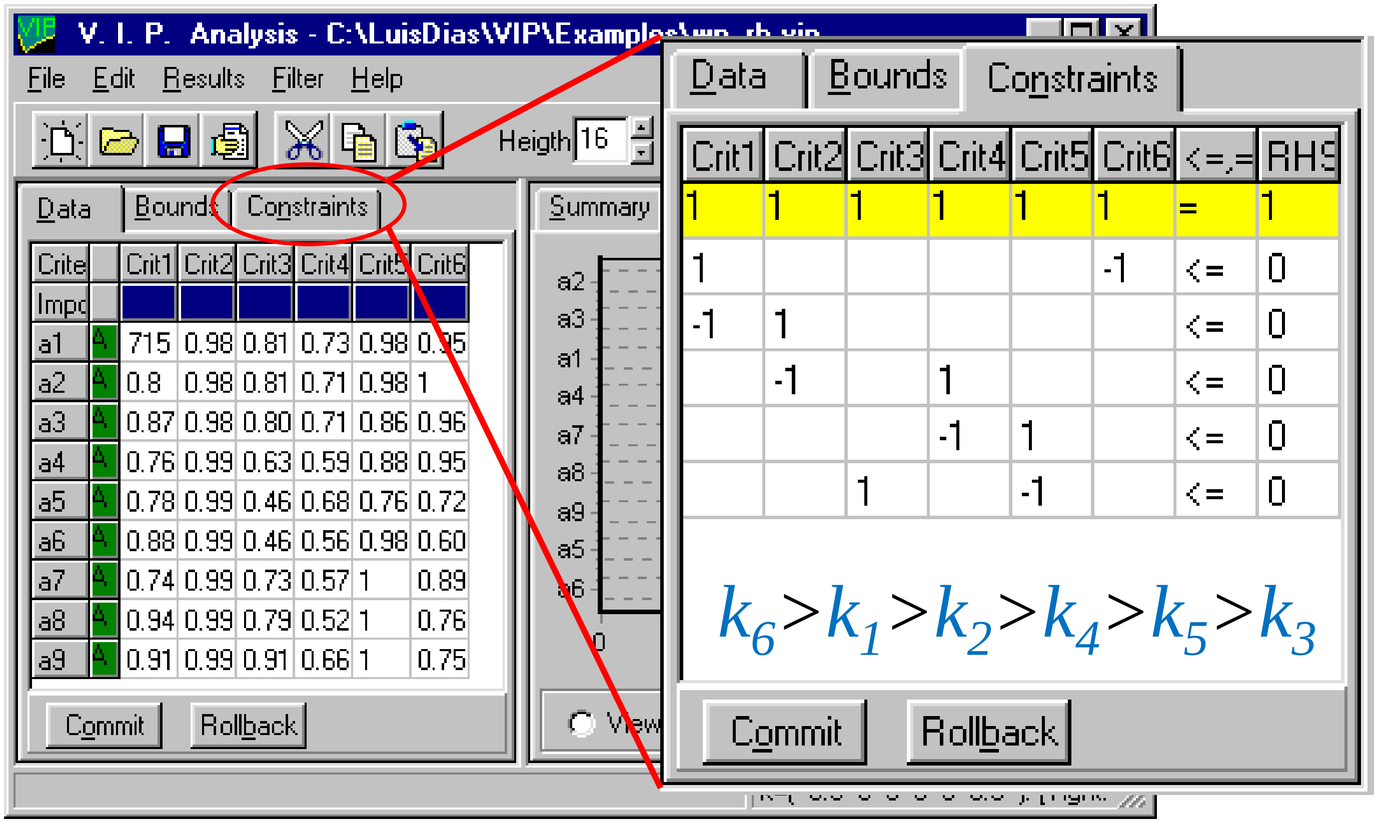
Weighted linear pool of experts

Additive aggregation with VIP Analysis

Dias, L. C., J. N. Clímaco, Additive Aggregation with Variable Interdependent Parameters: the VIP Analysis Software, *Journal of the Operational Research Society* 51, 1070-1082, 2000.

$$V(a_x) = k_1.v_1(a_x) + k_2.v_2(a_x) + \ldots + k_n.v_n(a_x),$$

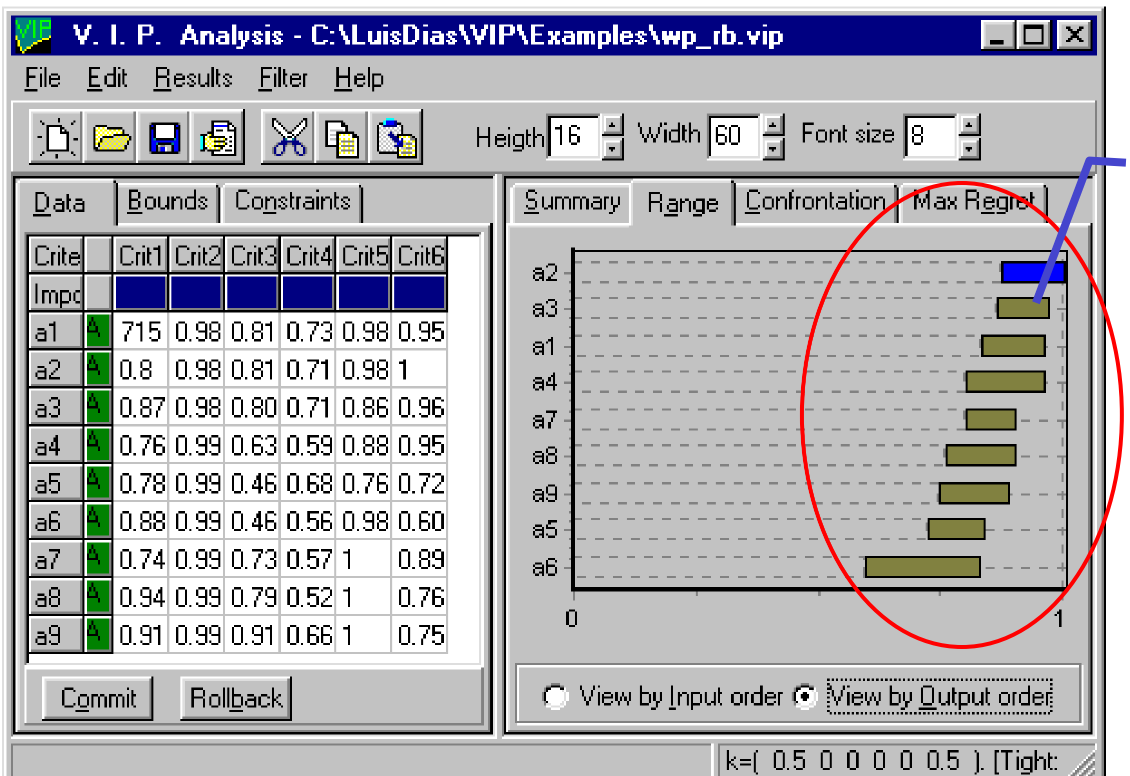
with $k \in T$ (set of admitted parameter values)



Additive aggregation with VIP Analysis

Minimum/maximum value for each alternative.

$$min\ \{V(a_x) : (k_1,\ldots,k_n) \in T\},\ max\ \{V(a_x) : (k_1,\ldots,k_n) \in T\}.$$

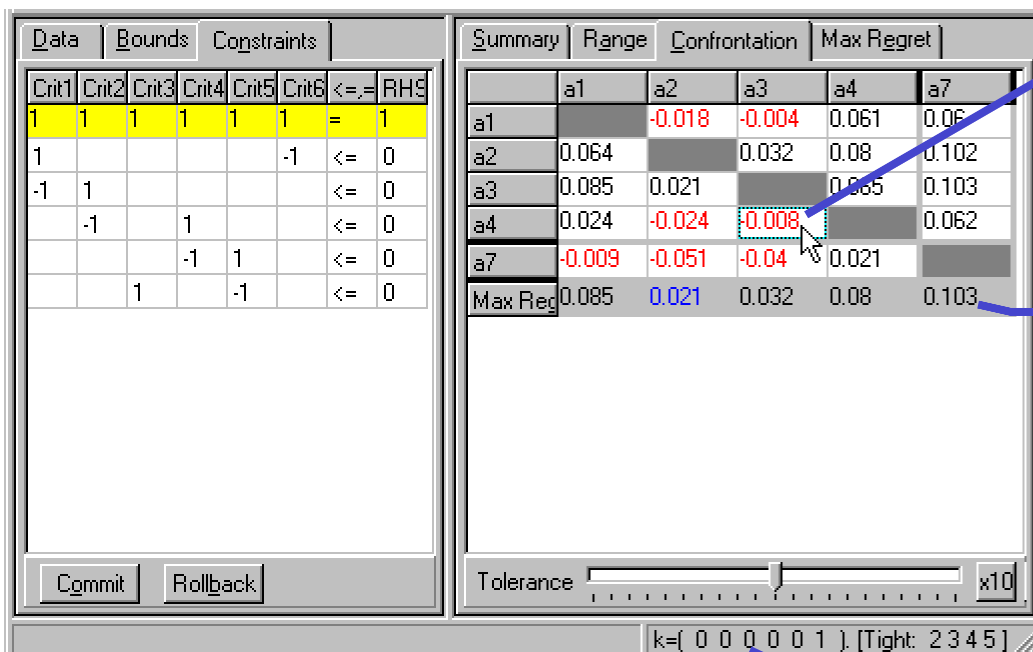


Range of possible results: robust conclusions about minimum and maximum value

Additive aggregation with VIP Analysis

Maximum advantage of a_x over a_y

$$max\{V(a_x)\text{-}V(a_y): k\in T\}$$



Relative dominance (e.g., a_4 dominated by a_3)

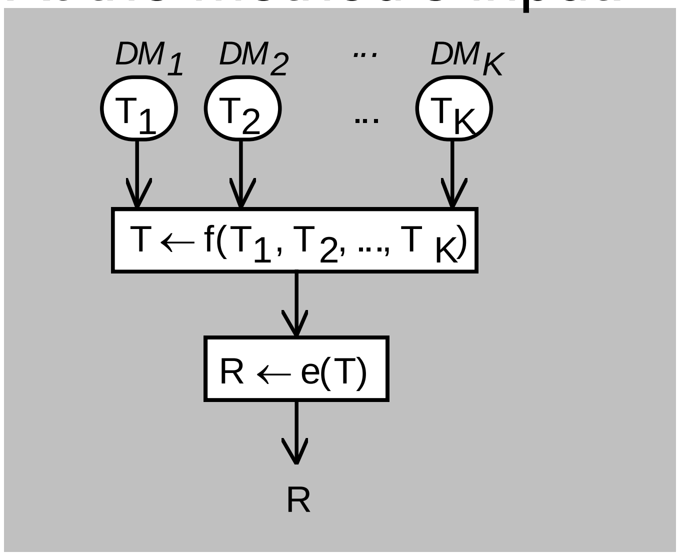
Robust conclusions about difference of value

Which parameter vector leads to an extreme difference?

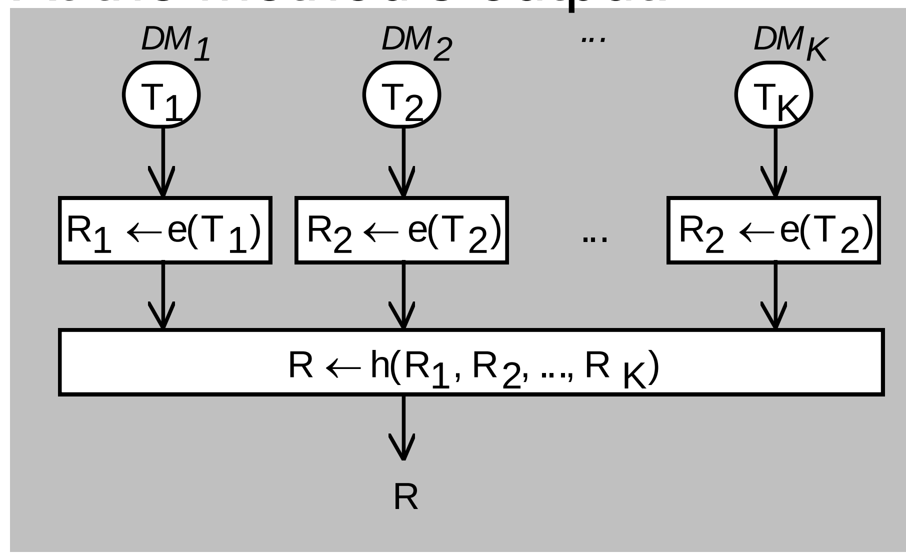
Group decision processes: Aggregation level

Dias, L.C., J.N. Clímaco, Dealing with imprecise information in group multicriteria decisions: A methodology and a GDSS architecture, *European Journal of Operational Research* 160 (2), 291-307, 2005

At the method's input:



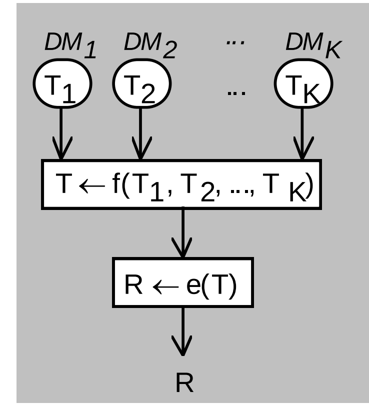
At the method's output:



The spirit behind the aggregation may be:

- to yield a result (voting, averaging, distance analysis)
- to provide each individual member with a reflection of the group's current inputs → ∪ and ∩ as operators

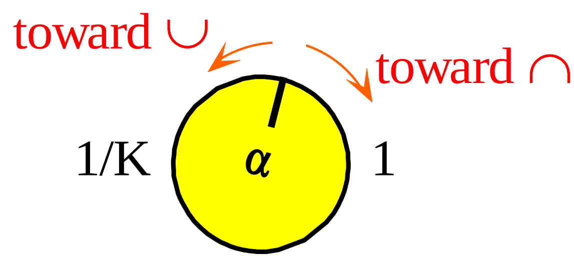
Aggregation at the inputs level



"majority level" $\alpha \in [1/K, 1]$:

$$T_{(\alpha)} = f_\alpha(T_1,\ldots,T_K) = \left\{ t \in \bigcup_{k=1}^{K} T_k : \frac{\#\{k \in \{1,\ldots,K\} : t \in T_k\}}{K} \geq \alpha \right\}$$

Inputs accepted by at least one DM

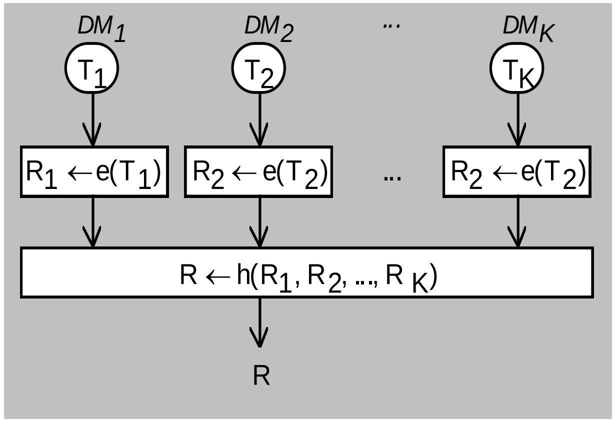


Inputs accepted by all the DMs

$$f_{1/K}(T_1,\ldots,T_K) = \bigcup_{k=1}^{K} T_k$$

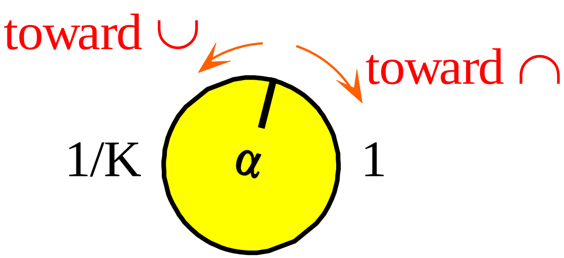
$$f_1(T_1,\ldots,T_K) = \bigcap_{k=1}^{K} T_k$$

Aggregation at the outputs level



$$R_{(\alpha)} = h_\alpha(R_1,\dots,R_K) = \left\{ r \in \bigcup_{k=1}^{K} R_k : \frac{\#\{k \in \{1,\dots,K\} : r \in R_k\}}{K} \geq \alpha \right\}$$

Outputs obtained by at least one DM



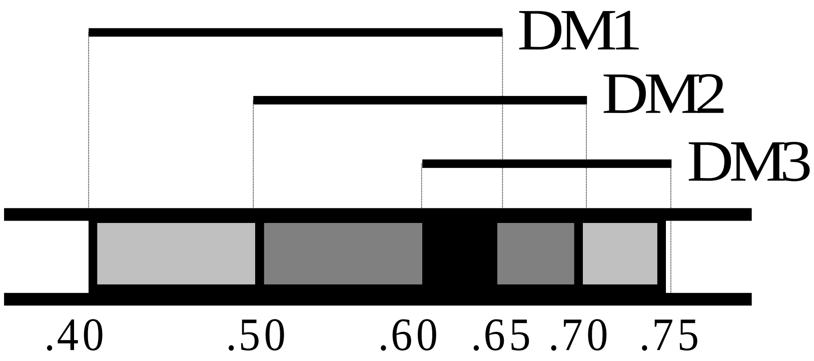
Outputs common to all the DMs

$$h_{1/K}(R_1,\dots,R_K) = \bigcup_{k=1}^{K} R_k$$

$$h_1(R_1,\dots,R_K) = \bigcap_{k=1}^{K} R_k$$

Duality between robustness and majority

An example (3 DMs):



$T_1 \rightarrow V(a_i) \in [0.40, 0.65]$

$T_2 \rightarrow V(a_i) \in [0.50, 0.70]$

$T_3 \rightarrow V(a_i) \in [0.60, 0.75]$

$V(a_i)_{(1/3)}$= [0.40, 0.75], i.e., $V(a_i) \geq 0.4$ has support of 3/3

$V(a_i)_{(3/3)}$= [0.60, 0.65], i.e., $V(a_i) \geq 0.6$ has support of 1/3

Compromise between robustness and majority

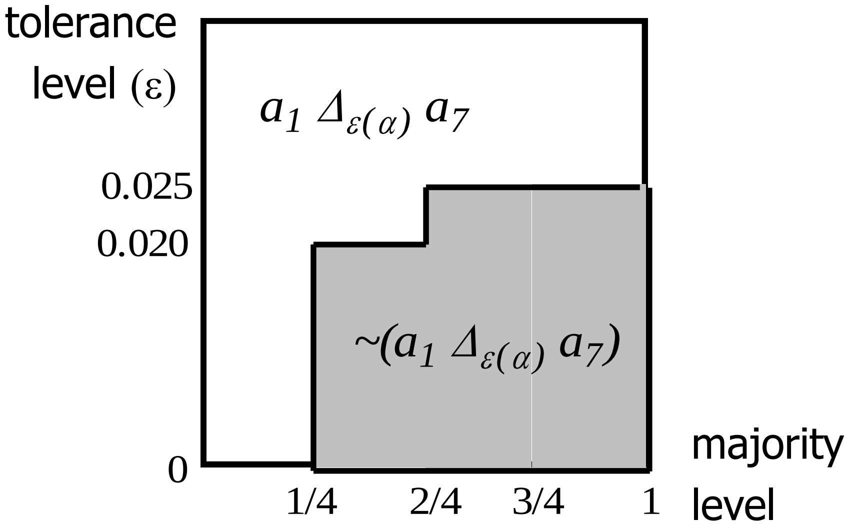
An example (4 DMs checking whether a_1 dominates a_7):

$T_1 \rightarrow max\{V(a_7)-V(a_1)\} = -0.01 \rightarrow a_1\ \Delta_0\ a_7$

$T_2 \rightarrow max\{V(a_7)-V(a_1)\} = 0.02 \rightarrow a_1\ \Delta_{0.020}\ a_7$

$T_3 \rightarrow max\{V(a_7)-V(a_1)\} = 0.025 \rightarrow a_1\ \Delta_{0.025}\ a_7$

$T_4 \rightarrow max\{V(a_7)-V(a_1)\} = 0.025 \rightarrow a_1\ \Delta_{0.025}\ a_7$



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Summing up

To begin with elicitation questions the DMs can answer *comfortably*,

progressively enriching the information as needed ("requisiteness" (Phillips, 1984) as stopping criterion),

using RA to guide the process (and see "where we're going"), to unveil robust conclusions, to motivate questions.

In group decision aiding

The purpose of aggregating individual models is not to obtain a solution automatically,

but rather to reflect to each group member the consequences of his/her inputs,

confronting them with analogous reflections of the group members' inputs.

Each group member can study what is robust from his/her perspective and from a group perspective.

Open to debate

Pros and cons of avoiding (or postponing) elicitation effort and conflict?

Can these ideas be of interest to elicit forecasts or probabilities (instead of preferences)?