

# Monitoring Remaining Project Completion Uncertainty, a Bayes Network Approach



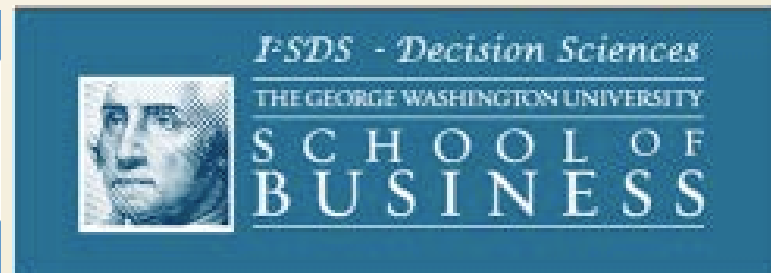
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**Johan Rene van Dorp and Ifechukwu Nduka**

Department of Engineering Management and Systems Engineering

*Sponsored By:*

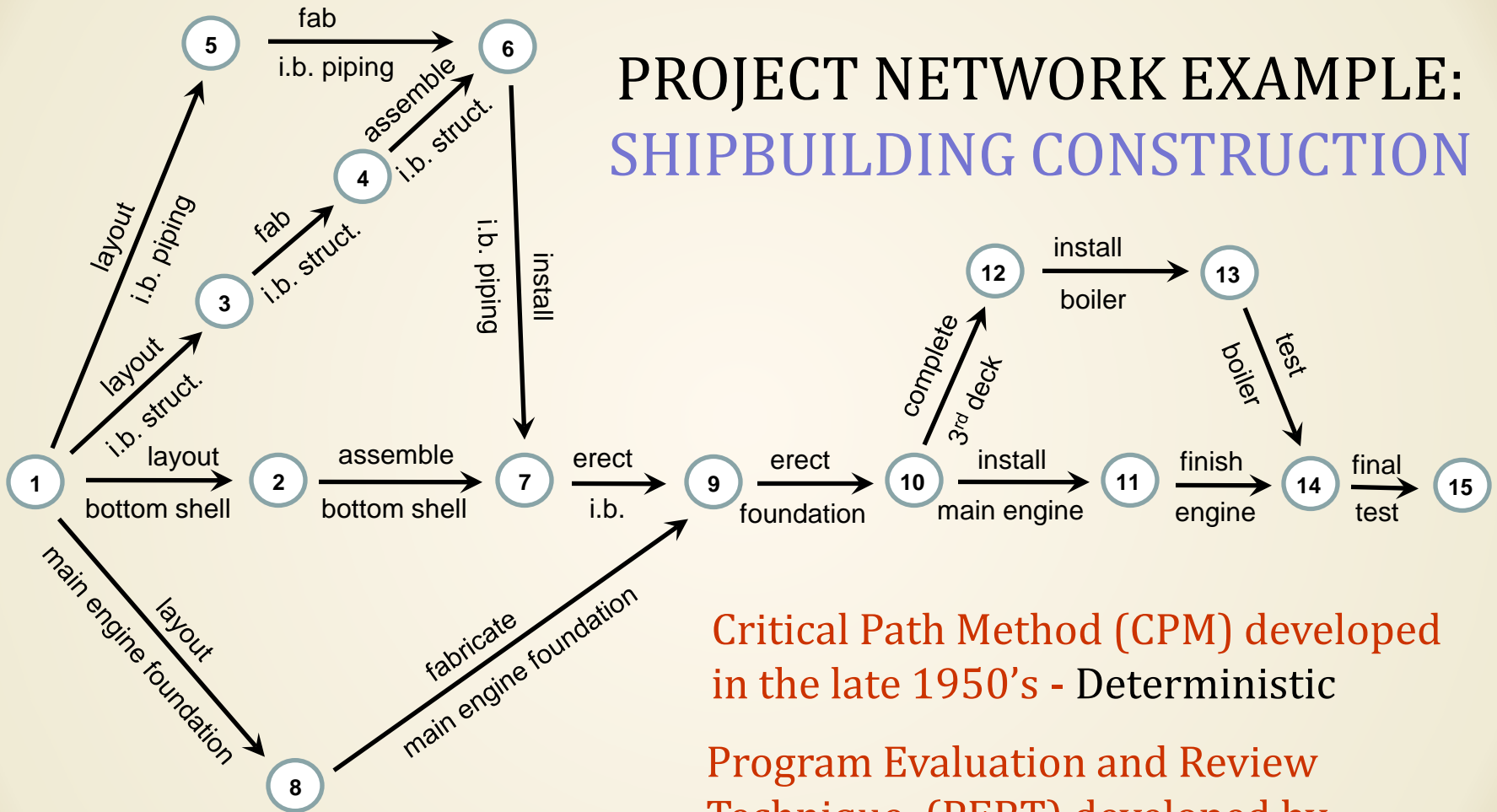


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- Case Study Description
- Bayesian Network Model
- Statistical Dependence Elicitation
- Uncertainty ( $n$ ) and Dependence ( $\beta$ ) Parameter Selection
- Prior Completion Time Uncertainty
- Posterior Analysis: Monitoring Uncertainty
- Conclusion

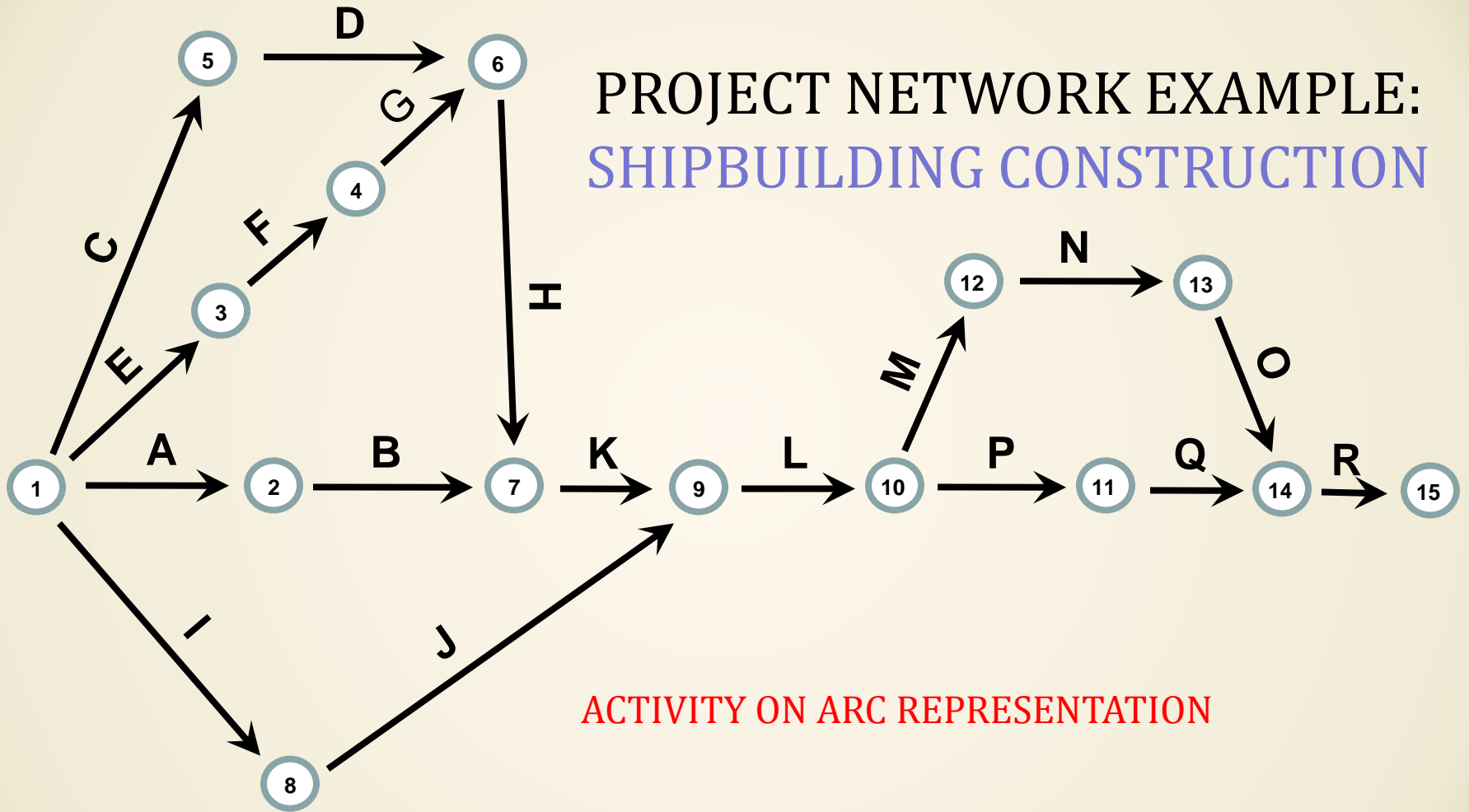
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



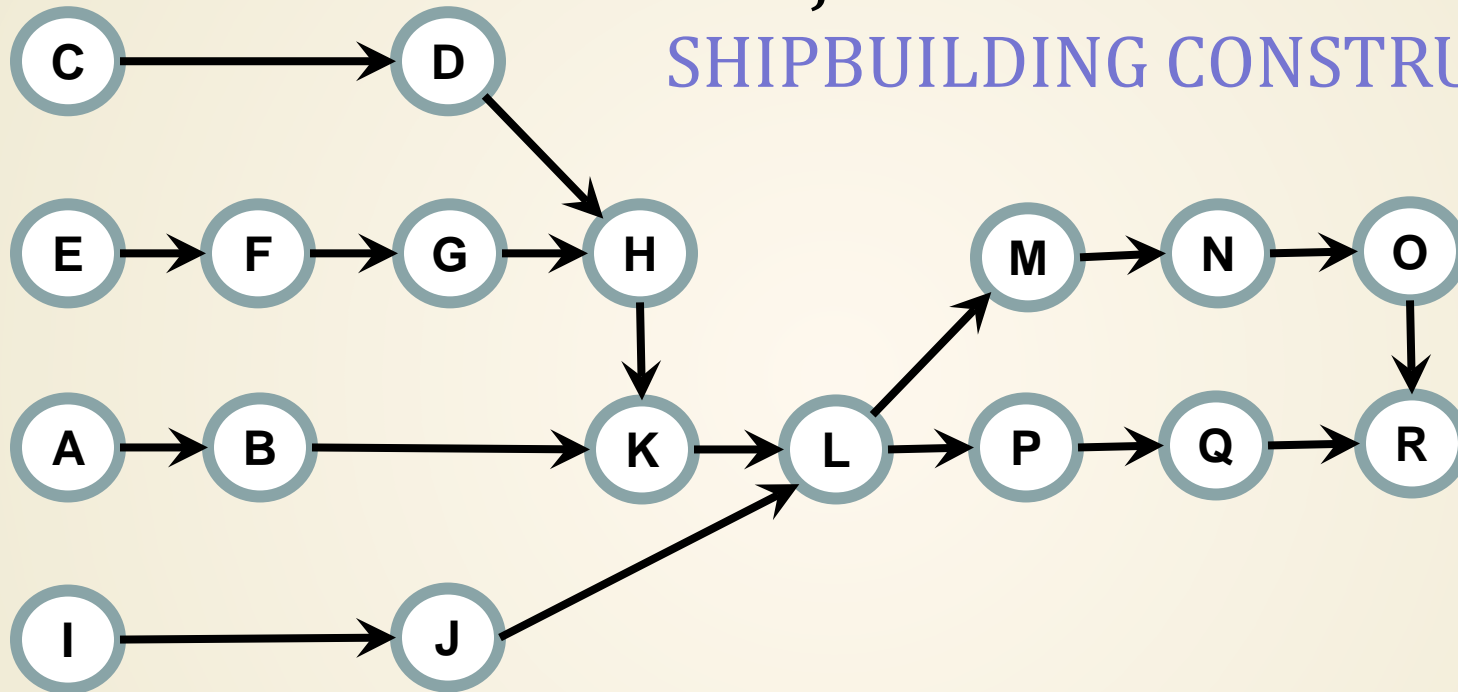
Critical Path Method (CPM) developed  
in the late 1950's - Deterministic

Program Evaluation and Review  
Technique (PERT) developed by  
Malcolm et al. (1959) - Stochastic CPM

# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



ACTIVITY ON NODE REPRESENTATION

# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION

ID	a	m	b	$\delta$	$C(\delta)$	PERT Variance	Modified PERT Variance
A	22	25	30	0.375	1.250	1.778	2.222
B	35	37	43	0.250	1.143	1.778	2.032
C	19	22	29	0.300	1.194	2.778	3.317
D	4	5	10	0.167	1.032	1.000	1.032
E	23	26	31	0.375	1.250	1.778	2.222
F	16	18	24	0.250	1.143	1.778	2.032
G	11	14	20	0.333	1.222	2.250	2.750
H	6	7	12	0.167	1.032	1.000	1.032
I	25	28	33	0.375	1.250	1.778	2.222
J	33	35	40	0.286	1.181	1.361	1.607
K	27	30	37	0.300	1.194	2.778	3.317
L	6	7	11	0.200	1.080	0.694	0.750
M	4	5	9	0.200	1.080	0.694	0.750
N	6	7	10	0.250	1.143	0.444	0.508
O	9	10	15	0.167	1.032	1.000	1.032
P	6	7	12	0.167	1.032	1.000	1.032
Q	17	20	26	0.333	1.222	2.250	2.750
R	13	15	20	0.286	1.181	1.361	1.607
Average Variance						1.790	

Malcolm  
et al. 1959

Herrerias  
et al. 2011

Traditional Activity Estimates

$$\text{PERT MEAN} = \frac{a + 4m + b}{6}$$

$$\text{PERT VARIANCE} = \frac{(b-a)^2}{36}$$

$$\text{MOD. PERT VARIANCE} = C(\delta) \times \frac{(b-a)^2}{36}$$

$$\delta = \frac{m-a}{b-a}, C(\delta) = \frac{5}{7} + \frac{16}{7} \times \delta(1-\delta)$$

# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION

Malcolm et al. (1959) assumed statistical independence between activity durations

This is a specious assumption however!

- Imagine activities that have to be completed under the open sky  
Imagine activities that require the use of a single crane  
Imagine activities completed by the same subcontractor
- Approaches thus far that have tried to relax the independence assumption unfortunately suffer from high parameter specification burden in a problem context that already suffers from that “ailment”
- Looking to relax that independence assumption in a “pragmatic” manner that builds on Malcolm’s et al. (1959) work



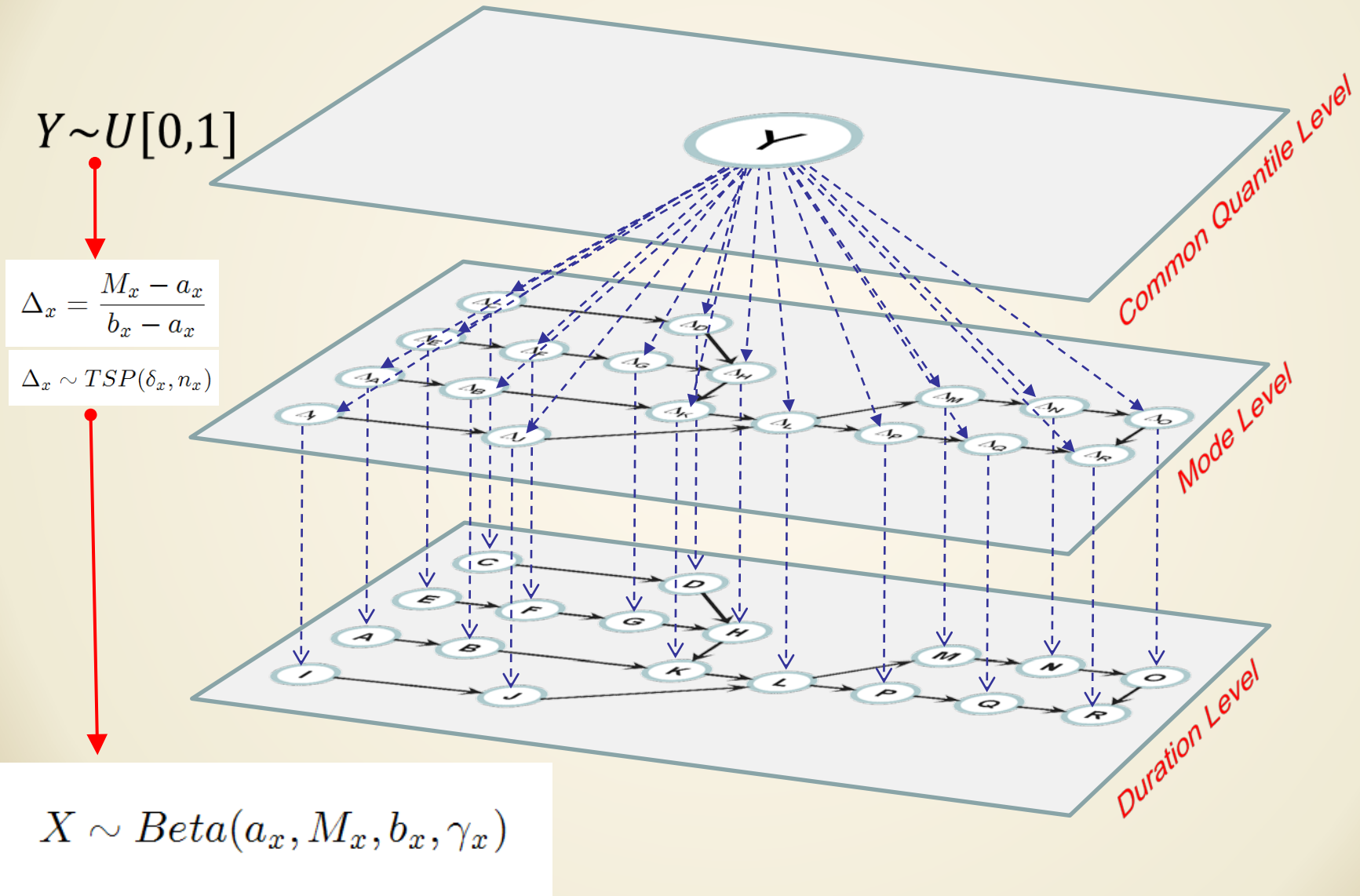
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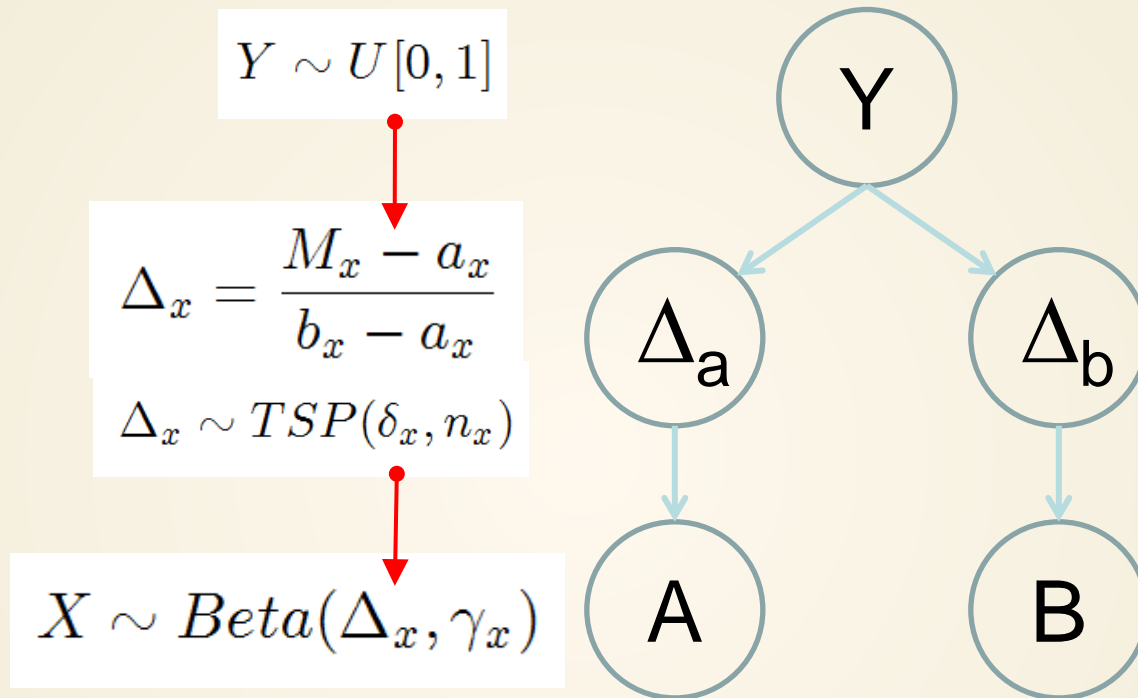
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- **Bayesian Network Model**
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# Bayesian Network Dependence Model for Project Risk Analysis



# How does dependence materialize throughout the Bayesian Network?



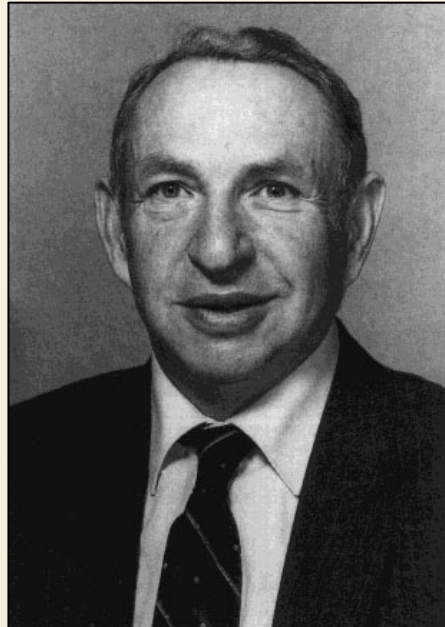
## Mode parameterization of Beta Distribution

$$g_X(x|\Delta_x, \gamma_x) = \frac{x^{\gamma_x \Delta_x} (1-x)^{\gamma_x (1-\Delta_x)}}{\mathbb{B}(\gamma_x \Delta_x + 1, \gamma_x (1-\Delta_x) + 1)}$$

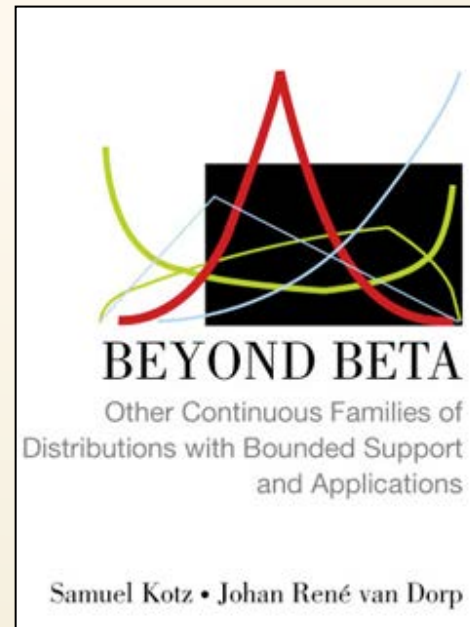
# How does dependence materialize throughout the Bayesian Network?

## Two Sided Power Density of Mode Relative Distance $\Delta_x$

$$f_{\Delta_x}(u|\delta_x, n_x) = n_x \times \begin{cases} \left(\frac{u}{\delta_x}\right)^{n_x-1}, & 0 \leq u \leq \delta_x, \\ \left(\frac{1-u}{1-\delta_x}\right)^{n_x-1}, & \delta_x \leq u \leq 1. \end{cases}$$



S. Kotz 1930 – 2010



2004

# Some Two-Sided Power Distribution Properties

Cumulative Distribution Function:

$$F_{\Delta_x}(u|\delta_x, n_x) = \begin{cases} \delta_x \left(\frac{u}{\delta_x}\right)^{n_x}, & 0 \leq u \leq \delta_x, \\ 1 - (1 - \delta_x) \left(\frac{1-u}{1-\delta_x}\right)^{n_x}, & \delta_x \leq u \leq 1, \end{cases}$$

Quantile Function:

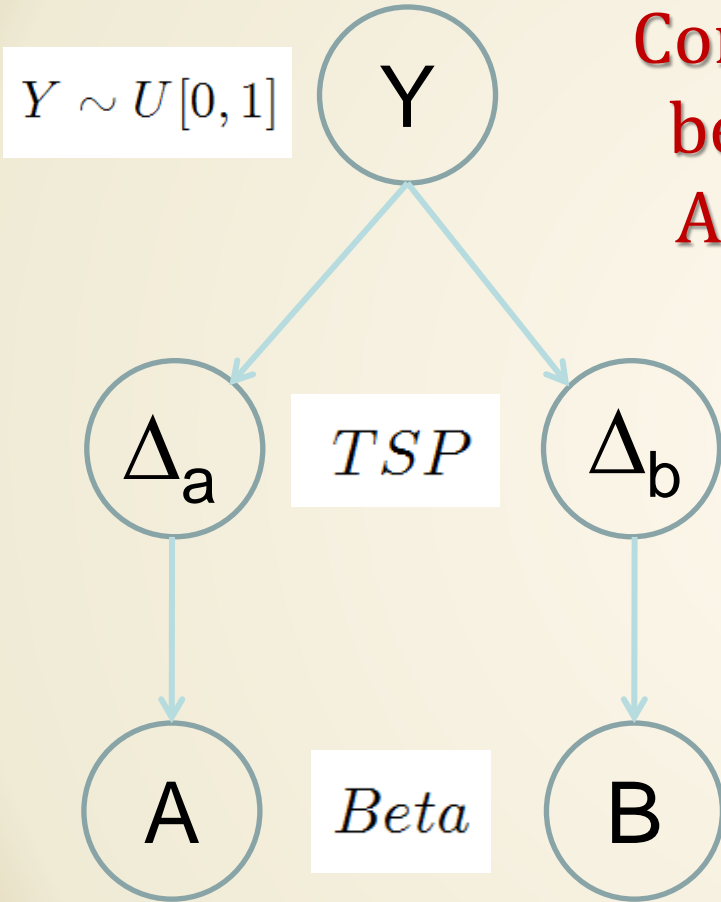
$$F_{\Delta_x}^{-1}(y|\delta_x, n_x) = \begin{cases} \delta_x \left(\frac{y}{\delta_x}\right)^{1/n_x}, & 0 \leq y \leq \delta_x, \\ 1 - (1 - \delta_x) \left(\frac{1-y}{1-\delta_x}\right)^{1/n_x}, & \delta_x \leq y \leq 1. \end{cases}$$

Conclusion:

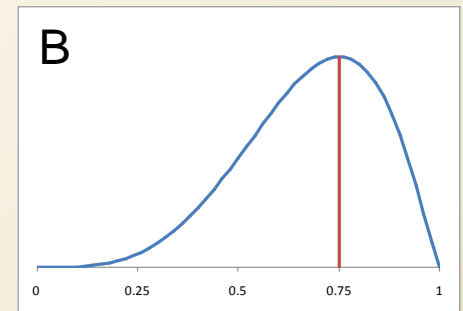
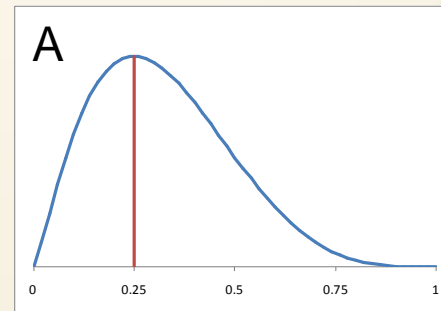
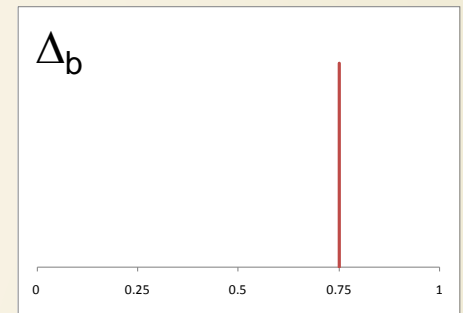
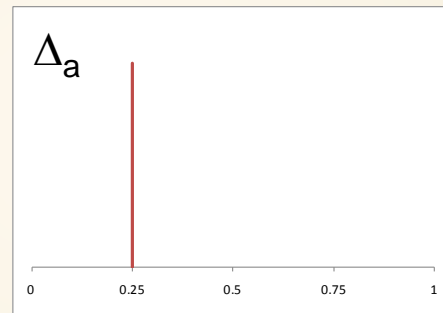
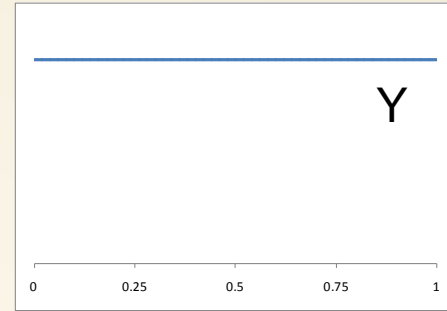
$$F_{\Delta_x}^{-1}(y|\delta_x, n_x) = F_{\Delta_x}(y|\delta_x, 1/n_x).$$

**QF and CDF are of the same functional form!**

# How does dependence materialize throughout this Bayesian Network?

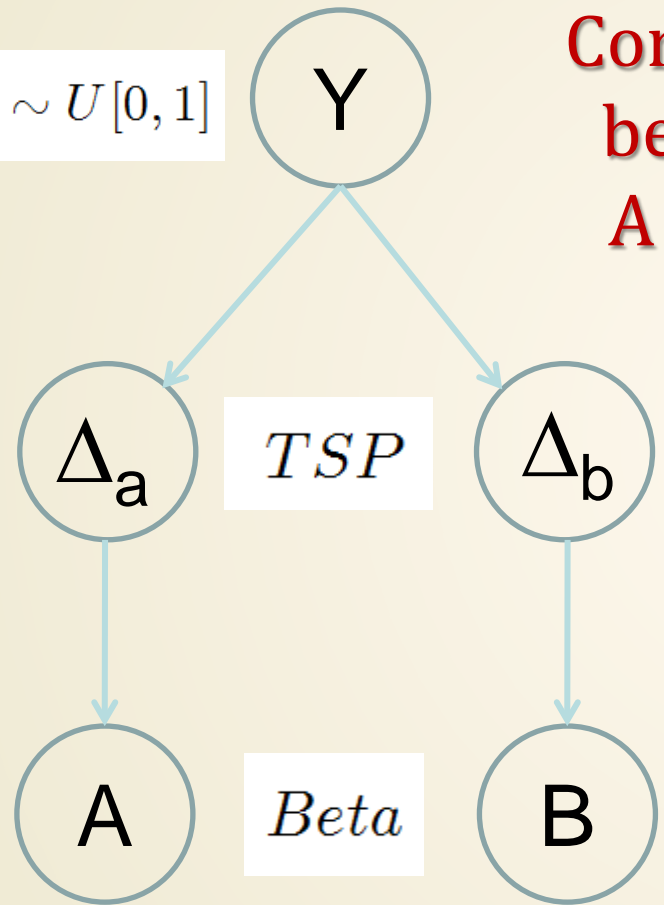


**Correlation  
between  
A and B?**

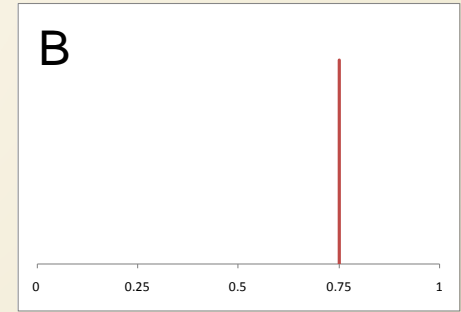
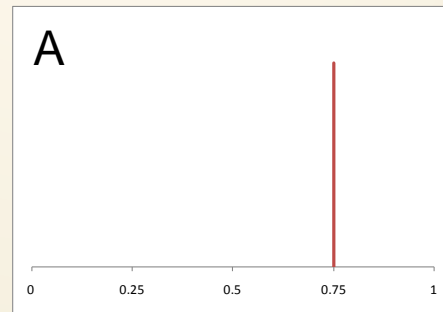
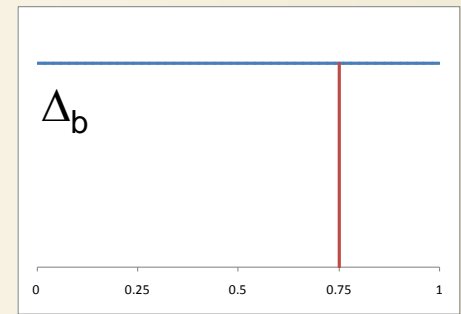
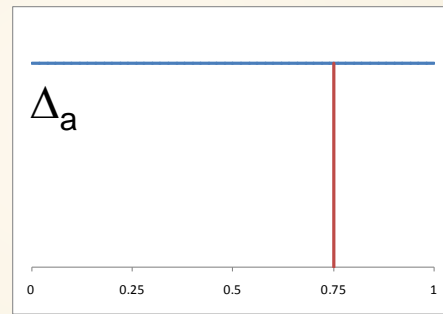
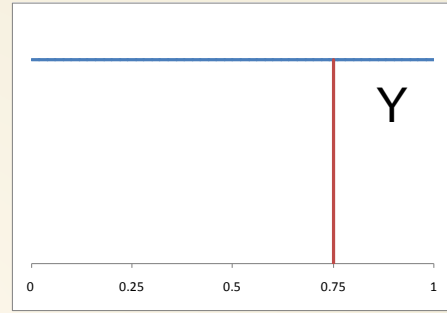


# How does dependence materialize throughout this Bayesian Network?

$$Y \sim U[0, 1]$$

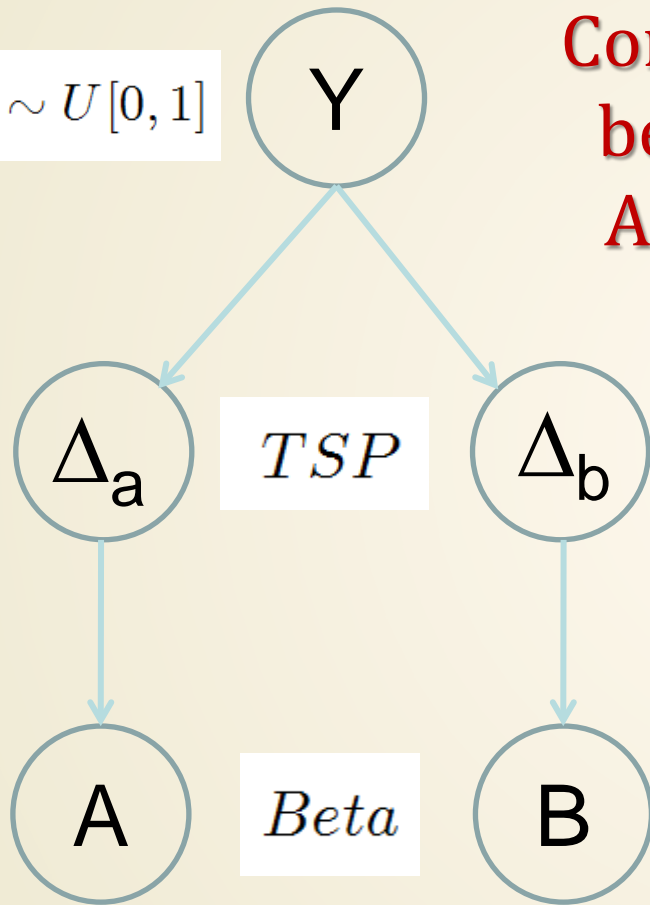


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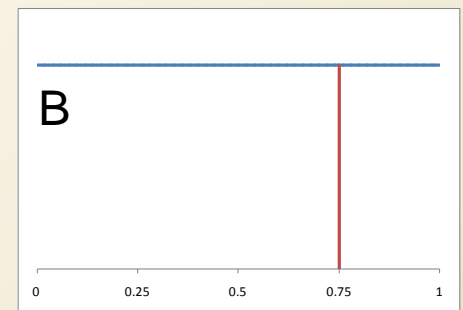
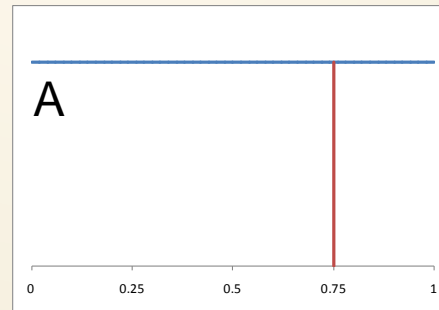
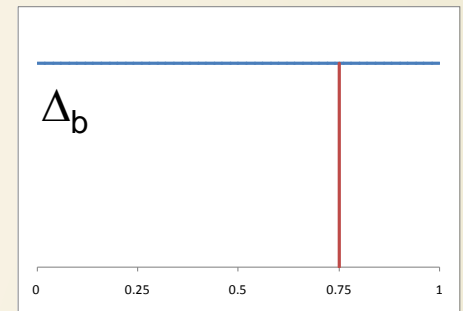
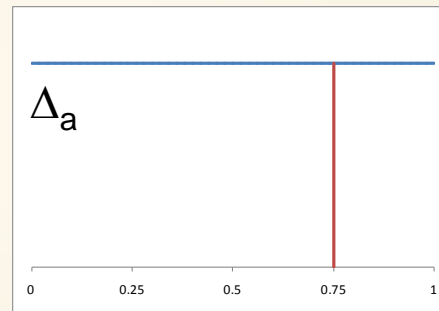
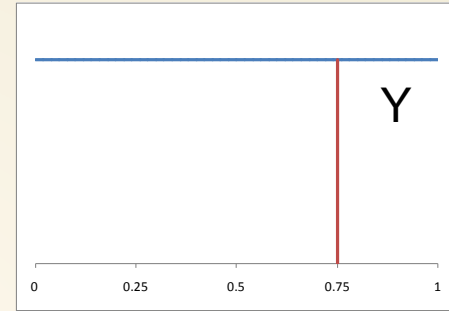


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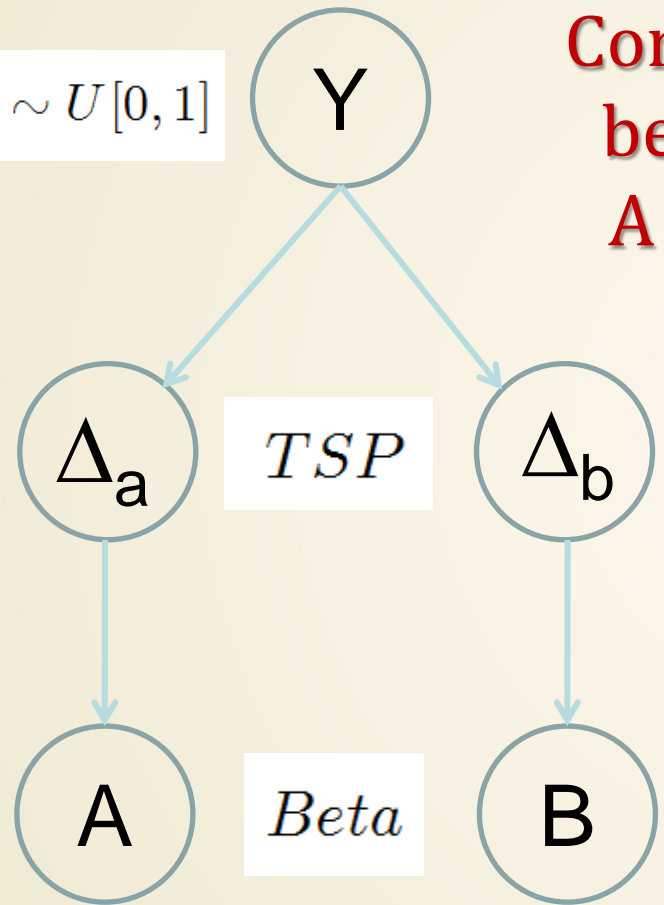
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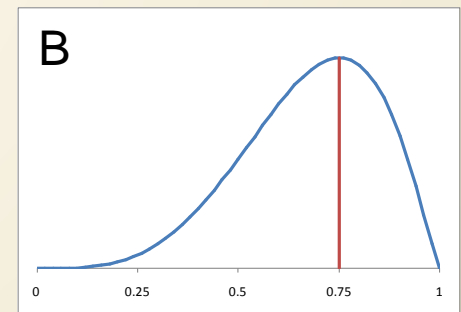
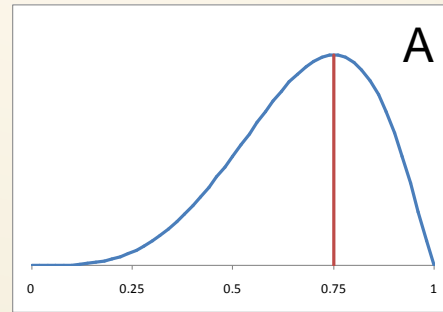
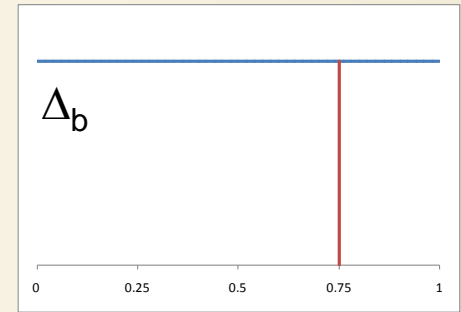
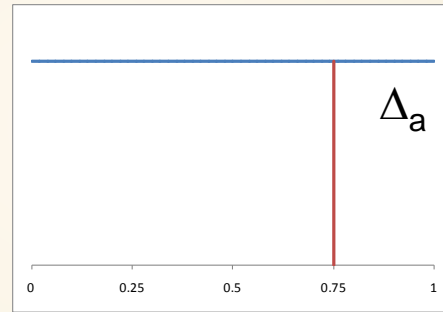
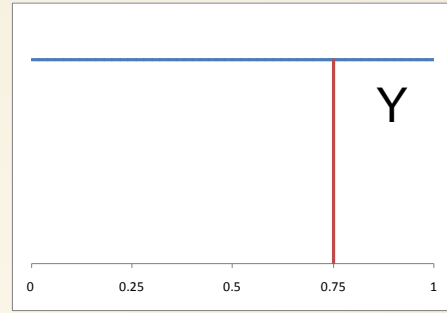


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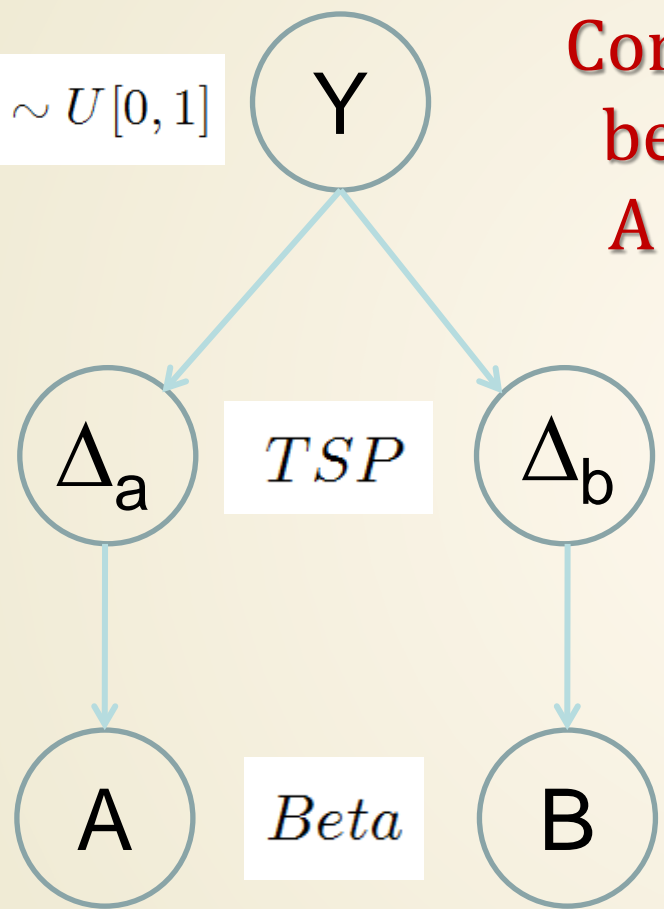


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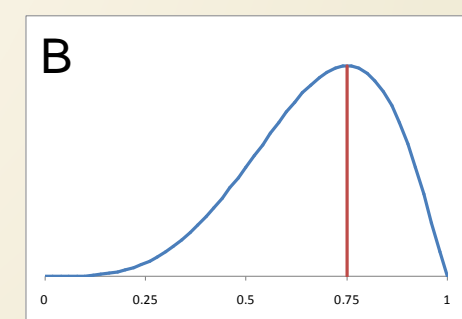
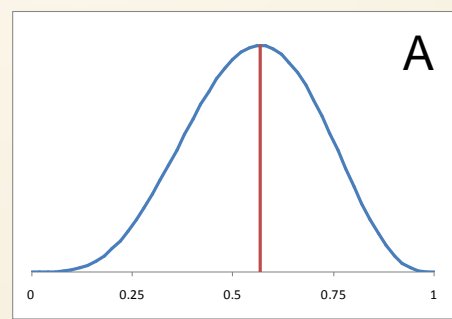
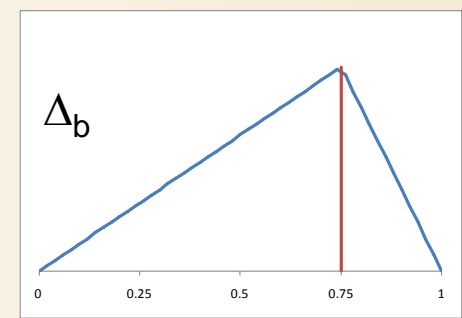
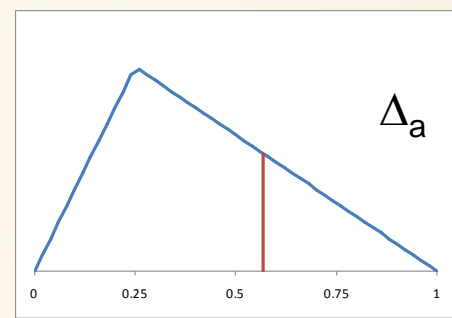
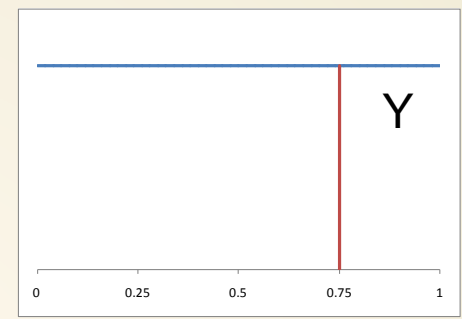


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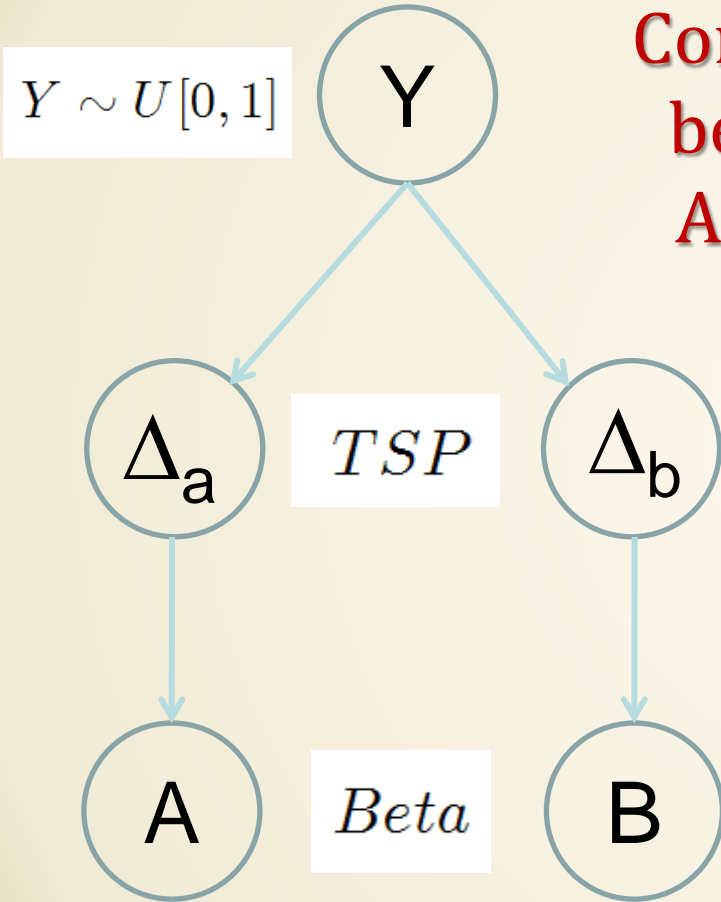
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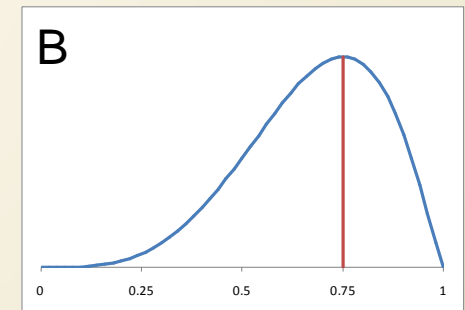
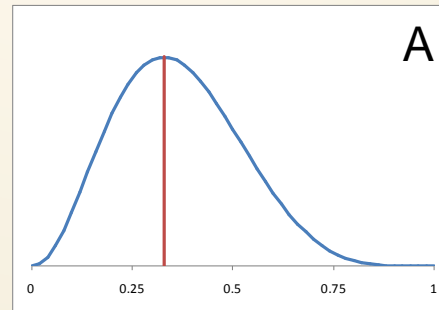
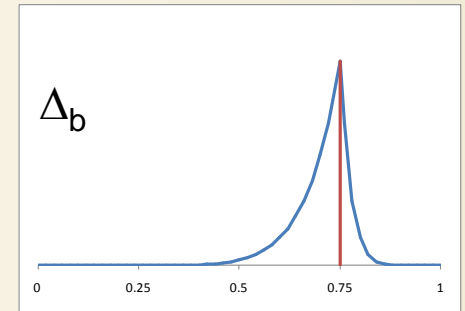
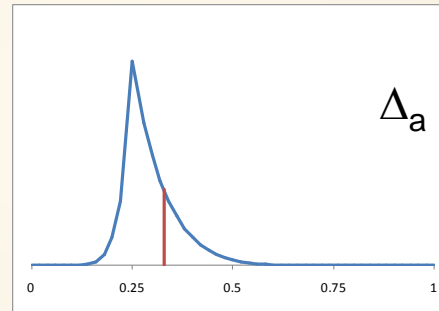
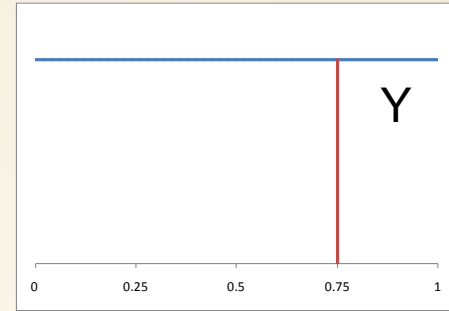
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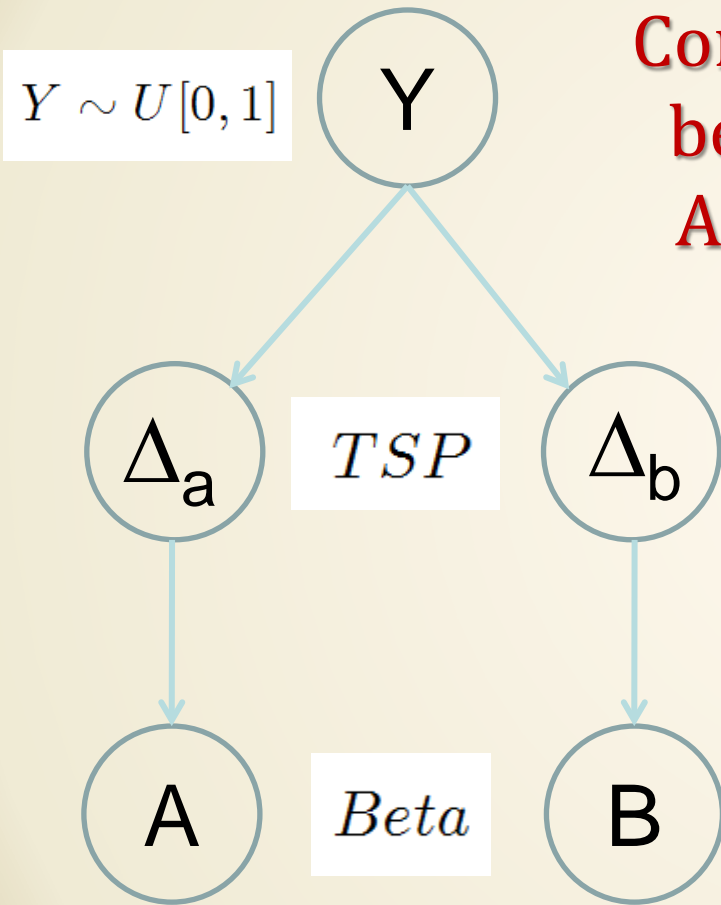
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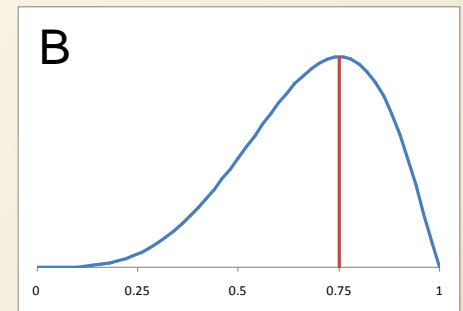
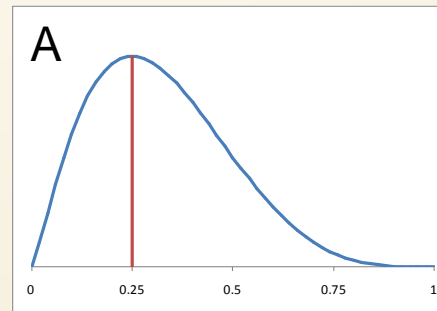
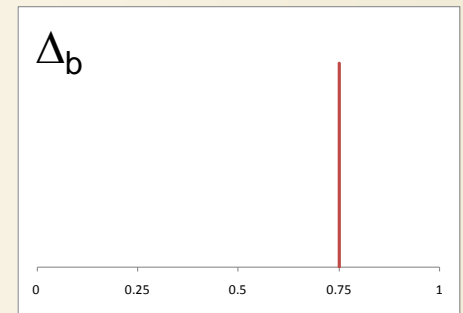
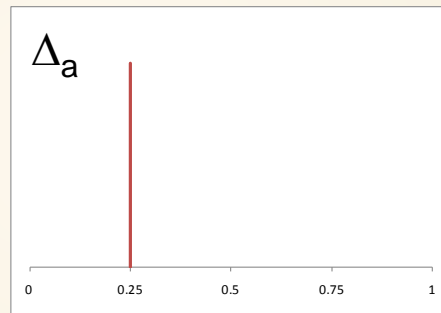
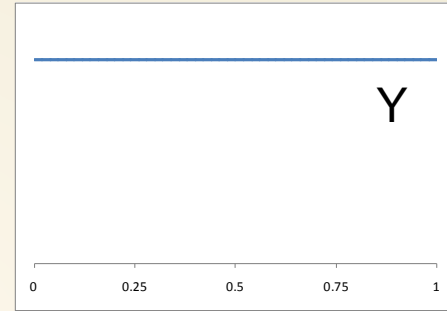
**Correlation  
between  
A and B?**



# How does dependence materialize throughout the Bayesian Network?



**Correlation  
between  
A and B?**



# How does dependence materialize throughout the Bayesian Network?

**Conclusion:** Assuming Activities share a common *TSP* shape parameter  $n$  and a common *Beta* shape parameter  $g$  we have:

- Keeping  $n$  constant **the correlation increases** with increasing  $\gamma$
- Keeping  $\gamma$  constant **the correlation decreases** with increasing  $n$

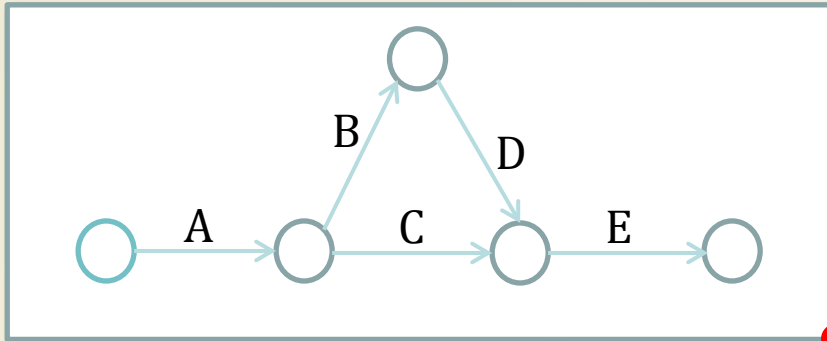
Assuming **common TSP shape parameter  $n$**  and **common Beta shape parameter  $\gamma$**  amongst activities, how does one specify them using expert elicitation **building on** Malcolm's et al. (1959) PERT method?

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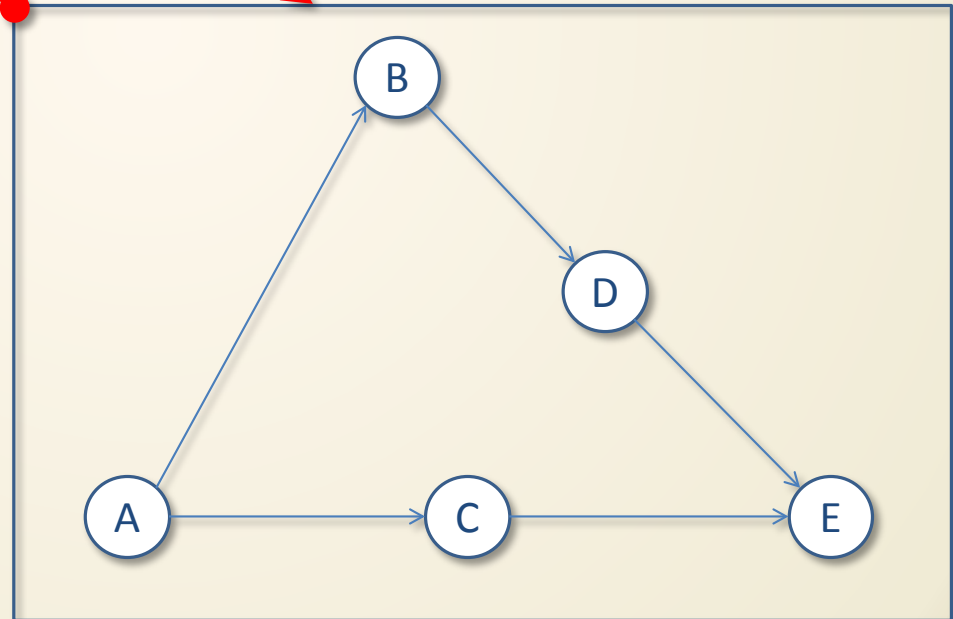
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# Small Activity Network Example



Activity-on-Arc  
Representation

Activity-on-Node  
Representation





# Eliciting Conditional Medians

“Suppose Activity A has finished above its median value  $a_{0.5}$ , what is the probability that Activity B finishes above its median value  $b_{0.5}$ ?”

If answer is “= 0.50”, knowledge of A does not influence B

If answer is “> 0.50”, knowledge of A does influence B

If answer is “= 1.00”, knowledge of A influences B “the most”

**Elicit activity conditional medians that are directly path-connected**

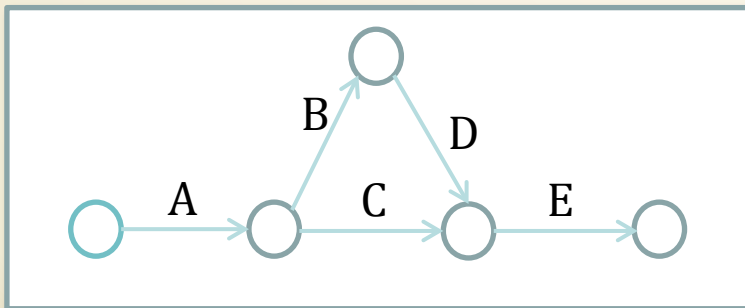
1.  $\Pr \{B > b_{0.5} | A > a_{0.5}\}$

2.  $\Pr \{C > c_{0.5} | A > a_{0.5}\}$

3.  $\Pr \{D > d_{0.5} | B > b_{0.5}\}$

4.  $\Pr \{E > e_{0.5} | C > c_{0.5}\}$

5.  $\Pr \{E > e_{0.5} | D > d_{0.5}\}$



# Conditional Median Matrix

Notation:  $\Pr \{ Y > y_{0.5} \mid X > x_{0.5} \} = \text{Med}(Y|X)$

	A	B	C	D	E
A	1	$\text{Med}(B A)$	$\text{Med}(C A)$	$\text{Med}(D A)$	$\text{Med}(E A)$
B	$\text{Med}(A B)$	1	$\text{Med}(C B)$	$\text{Med}(D B)$	$\text{Med}(E B)$
C	$\text{Med}(A C)$	$\text{Med}(B C)$	1	$\text{Med}(D C)$	$\text{Med}(E C)$
D	$\text{Med}(A D)$	$\text{Med}(B D)$	$\text{Med}(C D)$	1	$\text{Med}(E D)$
E	$\text{Med}(A E)$	$\text{Med}(B E)$	$\text{Med}(C E)$	$\text{Med}(D E)$	1

**Conditional Median Matrix is Symmetric:**

$$\begin{aligned}
 \text{Med}(X|Y) &= \Pr \{ X > x_{0.5} \mid Y > y_{0.5} \} = \frac{\Pr \{ Y > y_{0.5} \mid X > x_{0.5} \} \Pr \{ X > x_{0.5} \}}{\Pr \{ Y > y_{0.5} \}} \\
 &= \frac{\Pr \{ Y > y_{0.5} \mid X > x_{0.5} \} \times 0.5}{0.5} = \Pr \{ Y > y_{0.5} \mid X > x_{0.5} \} = \text{Med}(Y|X)
 \end{aligned}$$

# Conditional Median Matrix

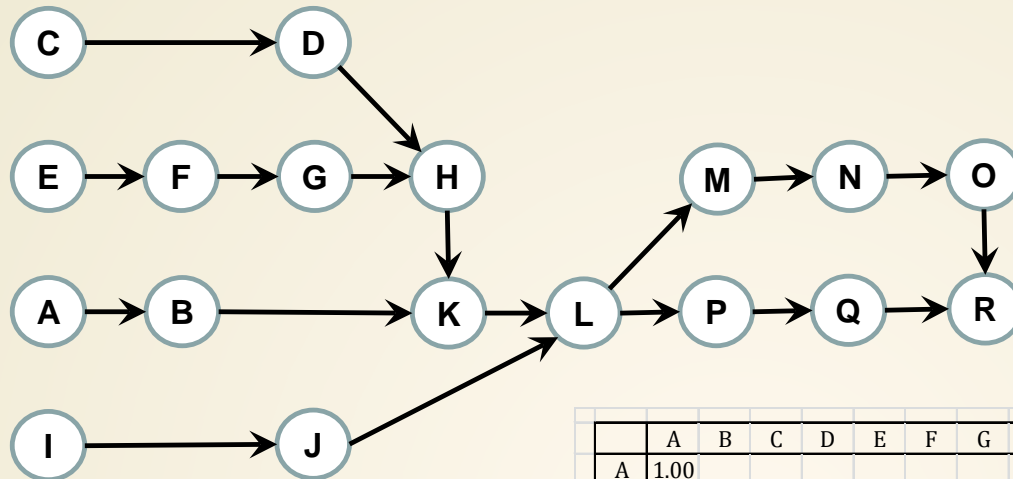
Notation:  $\Pr \{ Y > \text{Med}(Y) \mid X > \text{Med}(X) \} = \text{Med}(Y|X)$

	A	B	C	D	E
A	1	<b>Med(B A)</b>	<b>Med(C A)</b>	Med(D A)	Med(E A)
B		1	Med(C B)	<b>Med(D B)</b>	Med(E B)
C			1	Med(D C)	<b>Med(E C)</b>
D				1	<b>Med(E D)</b>
E					1

## Average Conditional Median

$$\overline{\text{Med}(\cdot|\cdot)} = \frac{\sum_{\substack{Y \in \{A,B,C,D,E\} \\ Y \neq X}} \sum_{X \in \{A,B,C,D,E\}} \text{Med}(Y|X)}{5 \times (5-1)}$$

# Back to Case Study



Eliciting only activity-to-activity conditional medians that are directly connected in Project Network reduces elicitation burden here from:

$$\binom{18}{2} \text{ to } 18 \text{ conditional medians}$$

153  
↑

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
A	1.00																	
B	0.65	1.00																
C			1.00															
D			0.65	1.00														
E					1.00													
F					0.70	1.00												
G						0.65	1.00											
H				0.70			0.60	1.00										
I									1.00									
J									0.70	1.00								
K		0.75									1.00							
L										0.65	0.60	1.00						
M												0.70	1.00					
N													0.55	1.00				
O														0.70	1.00			
P												0.65				1.00		
Q																0.65	1.00	
R															0.70	0.60	1.00	

Average Conditional Median  $\approx 0.658$

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- Case Study Description
- Bayesian Network Model
- Statistical Dependence Elicitation
- **Uncertainty ( $n$ ) and Dependence Parameter ( $\beta$ ) Selection**
- Prior Completion Time Uncertainty
- Posterior Analysis: Monitoring Uncertainty
- Conclusion

# Algorithmic Selection Procedure Parameters $\beta$ and $n$ :

## 1. For different values of $\beta$ and $n$ do:

- Sample 1000 joint activity samples from joint prior (defined by BN)
- Evaluate Average Prior Marginal Activity Variance
- Evaluate Average Prior Activity Statistical Dependence

## 2. Select values of $\beta$ and $n$ such that:

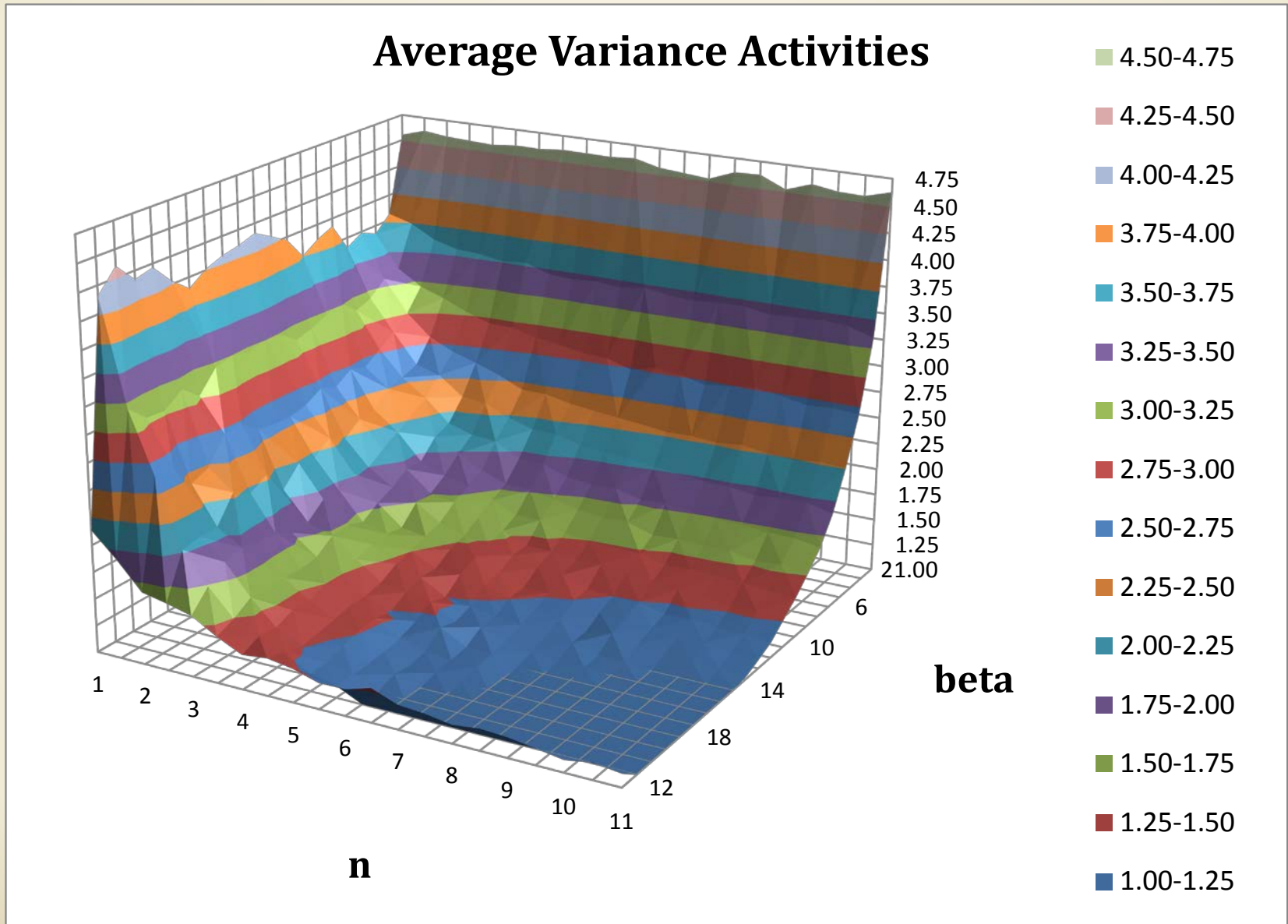
- Average Prior Marginal Activity Variance close to Average Modified PERT Variance across Project Network
- Average Prior Activity to Activity Statistical Dependence close to Elicited Average Activity to Activity Statistical Dependence

## 3. For selected values of $\beta$ and $n$ do:

- a. Sample 1000 Joint activities from Bayesian Network Prior
- b. Evaluate Average Prior Marginal Activity Variance
- c. Evaluate Average Prior Activity Statistical Dependence
- d. Repeat Steps a, b and c one 1000 times and plot:
  - Average Prior Marginal Activity Variance Uncertainty
  - Average Prior Activity to Activity Statistical Dependence Uncertainty
- e. Test if Average Modified PERT Variance and Elicited Average Statistical dependence fall within 90% credibility limits of uncertainty distributions



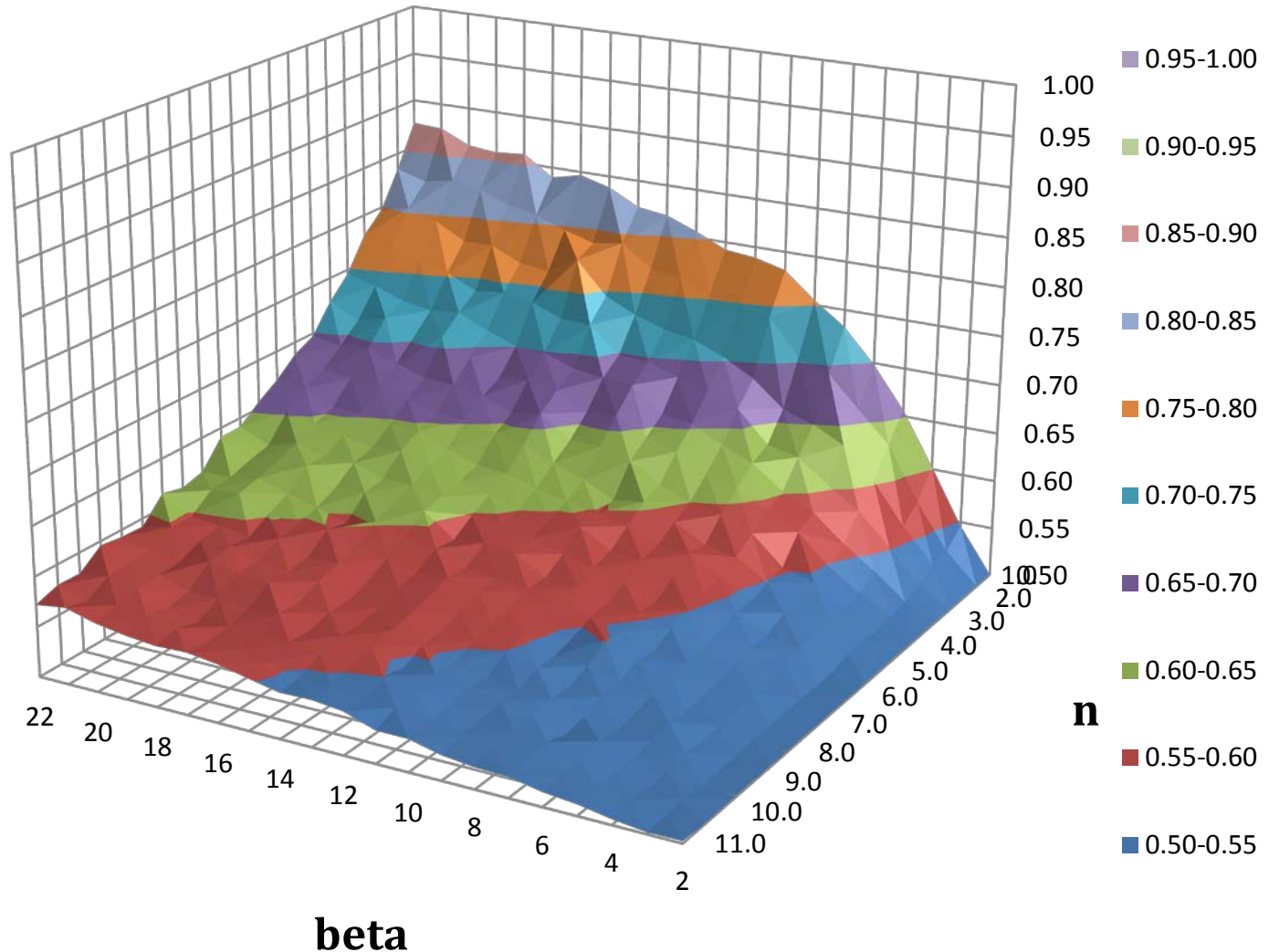
# STEP 1: Prior Average Variance ( $\beta$ , $n$ )





# STEP 1: Prior Average Activity Cond. Med( $\beta$ , $n$ )

## Average Activity Conditional Median



# STEP 2: Select values of $\beta$ and n that are close

$\beta$

Prior Average Activity Variance between 1.50 and 2.00

n

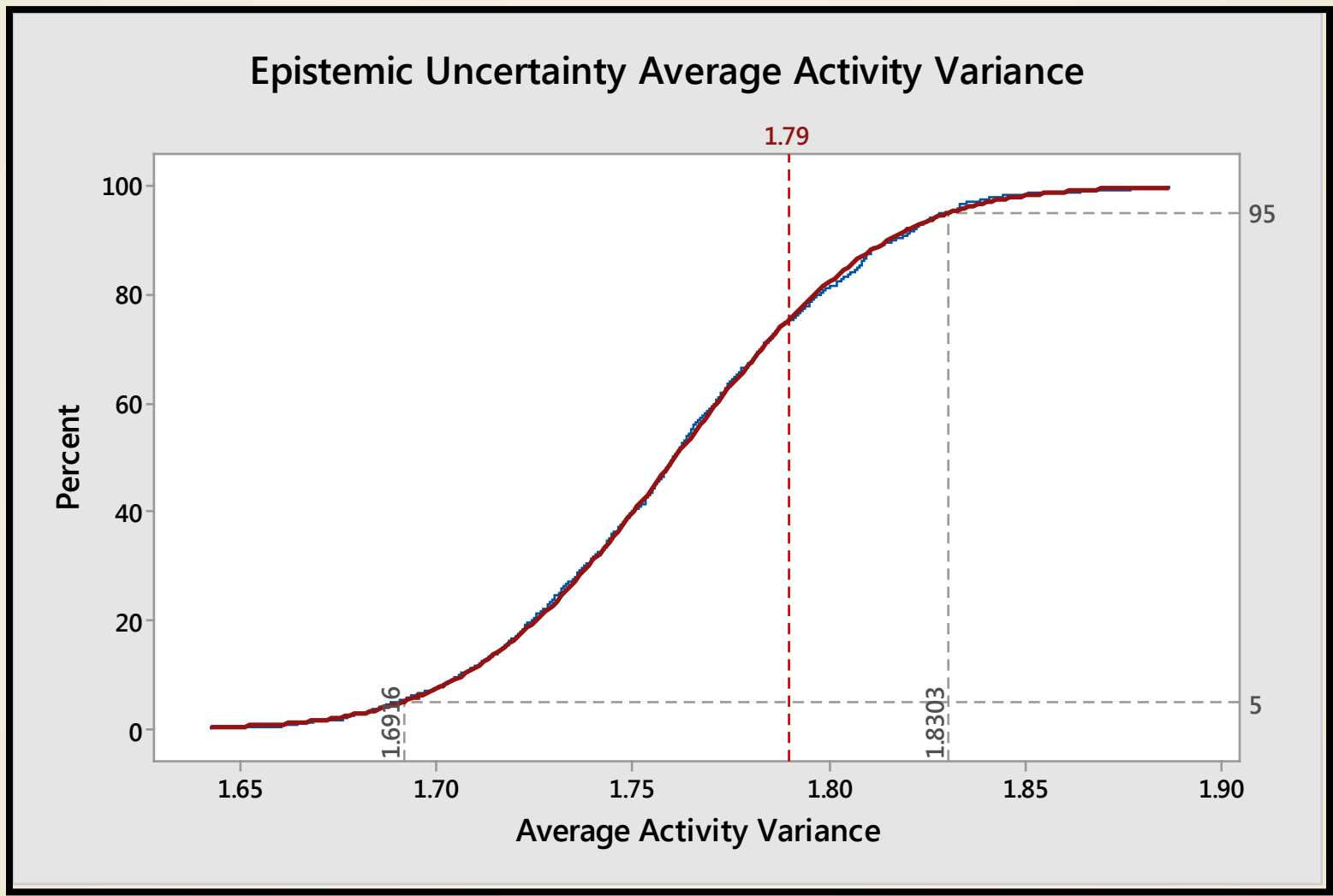
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1.0	4.53	3.80	3.65	3.59	3.66	3.69	3.77	3.96	3.83	3.78	4.00	3.98	3.97	4.13	4.19	3.93	4.30	4.26	4.16	4.28	4.31
1.5	4.54	3.69	3.40	3.34	3.23	3.23	3.10	3.17	3.19	3.12	3.10	3.04	3.22	3.09	3.14	3.12	3.18	3.17	3.20	3.17	3.11
2.0	4.56	3.61	3.19	3.01	2.90	2.83	2.59	2.61	2.75	2.67	2.56	2.57	2.52	2.72	2.53	2.53	2.49	2.50	2.61	2.61	2.62
2.5	4.61	3.62	3.15	2.87	2.68	2.52	2.40	2.42	2.35	2.33	2.30	2.17	2.16	2.13	2.22	2.13	2.13	2.03	2.11	2.15	2.12
3.0	4.65	3.57	2.99	2.64	2.49	2.37	2.25	2.15	2.09	2.04	2.06	2.00	1.87	1.89	1.83	1.75	1.78	1.74	1.84	1.79	1.78
3.5	4.58	3.56	2.94	2.64	2.42	2.25	2.12	1.96	1.95	1.85	1.93	1.69	1.74	1.73	1.65	1.60	1.69	1.58	1.59	1.52	1.56
4.0	4.62	3.51	2.91	2.54	2.28	2.13	2.01	1.84	1.73	1.75	1.71	1.55	1.61	1.57	1.51	1.46	1.38	1.40	1.40	1.35	1.35
4.5	4.57	3.48	2.88	2.46	2.19	2.07	1.89	1.75	1.73	1.53	1.56	1.49	1.48	1.45	1.39	1.35	1.35	1.31	1.27	1.23	1.25
5.0	4.54	3.46	2.86	2.49	2.15	2.00	1.84	1.68	1.64	1.56	1.44	1.36	1.35	1.30	1.27	1.28	1.22	1.21	1.10	1.14	1.14
5.5	4.62	3.44	2.82	2.43	2.13	1.91	1.78	1.69	1.53	1.47	1.37	1.33	1.31	1.24	1.21	1.18	1.11	1.12	1.11	1.07	1.06
6.0	4.60	3.46	2.83	2.39	2.08	1.90	1.67	1.61	1.48	1.41	1.34	1.27	1.23	1.21	1.14	1.10	1.09	1.05	1.02	0.99	1.00
6.5	4.55	3.37	2.80	2.27	2.07	1.88	1.70	1.56	1.46	1.35	1.31	1.23	1.16	1.18	1.11	1.08	1.04	1.07	0.96	0.91	0.98
7.0	4.61	3.41	2.75	2.35	2.05	1.87	1.67	1.52	1.40	1.35	1.23	1.20	1.17	1.09	1.09	1.01	0.99	0.99	0.97	0.89	0.88
7.5	4.54	3.41	2.78	2.31	2.01	1.77	1.67	1.48	1.40	1.30	1.22	1.19	1.12	1.04	1.03	1.00	0.96	0.91	0.92	0.89	0.85
8.0	4.55	3.46	2.74	2.30	2.01	1.77	1.62	1.45	1.39	1.26	1.20	1.12	1.08	1.01	1.00	0.93	0.94	0.87	0.85	0.84	0.83
8.5	4.58	3.41	2.69	2.30	1.95	1.73	1.59	1.45	1.35	1.24	1.14	1.13	1.05	1.01	0.94	0.91	0.89	0.86	0.82	0.80	0.77
9.0	4.57	3.38	2.79	2.30	1.95	1.74	1.57	1.42	1.28	1.21	1.16	1.10	1.00	0.97	0.94	0.93	0.87	0.85	0.80	0.77	0.76
9.5	4.63	3.39	2.73	2.29	1.91	1.70	1.55	1.40	1.31	1.24	1.14	1.06	1.02	0.96	0.93	0.85	0.83	0.78	0.78	0.80	0.78
10.0	4.55	3.43	2.71	2.27	1.95	1.71	1.53	1.42	1.29	1.19	1.13	1.03	1.01	0.95	0.89	0.85	0.82	0.77	0.77	0.76	0.73
10.5	4.59	3.37	2.73	2.24	1.95	1.70	1.51	1.36	1.26	1.17	1.09	1.01	0.94	0.91	0.89	0.85	0.81	0.79	0.76	0.72	0.69
11.0	4.63	3.45	2.67	2.24	1.96	1.68	1.53	1.36	1.28	1.16	1.10	1.02	0.96	0.93	0.88	0.85	0.79	0.75	0.76	0.73	0.69

Prior Average Conditional Median between 0.625 and 0.675

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1.0	0.50	0.55	0.60	0.66	0.70	0.73	0.75	0.78	0.79	0.79	0.81	0.82	0.82	0.84	0.85	0.84	0.86	0.86	0.86	0.87	0.88
1.5	0.50	0.53	0.58	0.63	0.66	0.69	0.71	0.74	0.75	0.75	0.77	0.77	0.79	0.79	0.80	0.81	0.81	0.83	0.83	0.82	0.83
2.0	0.50	0.53	0.56	0.60	0.63	0.65	0.66	0.69	0.70	0.72	0.72	0.73	0.75	0.78	0.76	0.77	0.78	0.78	0.80	0.80	0.80
2.5	0.50	0.52	0.55	0.58	0.60	0.63	0.63	0.66	0.68	0.69	0.70	0.70	0.72	0.73	0.74	0.74	0.74	0.75	0.76	0.77	0.78
3.0	0.50	0.51	0.54	0.56	0.59	0.60	0.62	0.64	0.65	0.66	0.68	0.68	0.69	0.70	0.70	0.71	0.73	0.72	0.73	0.74	0.75
3.5	0.50	0.51	0.53	0.55	0.58	0.59	0.61	0.62	0.63	0.64	0.66	0.66	0.68	0.68	0.68	0.69	0.71	0.70	0.72	0.71	0.73
4.0	0.50	0.51	0.52	0.54	0.55	0.58	0.59	0.60	0.61	0.63	0.64	0.64	0.66	0.66	0.67	0.67	0.68	0.69	0.70	0.69	0.70
4.5	0.50	0.51	0.52	0.54	0.55	0.57	0.57	0.59	0.61	0.60	0.62	0.63	0.64	0.64	0.65	0.66	0.66	0.67	0.67	0.68	0.69
5.0	0.50	0.51	0.52	0.54	0.54	0.56	0.58	0.58	0.59	0.60	0.60	0.61	0.62	0.62	0.63	0.65	0.65	0.65	0.65	0.67	0.67
5.5	0.50	0.50	0.52	0.53	0.54	0.55	0.56	0.58	0.58	0.60	0.60	0.60	0.61	0.62	0.63	0.64	0.63	0.64	0.65	0.65	0.66
6.0	0.50	0.50	0.52	0.53	0.54	0.55	0.55	0.56	0.57	0.58	0.59	0.59	0.60	0.61	0.62	0.62	0.63	0.63	0.64	0.63	0.64
6.5	0.50	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.57	0.58	0.59	0.59	0.61	0.61	0.61	0.61	0.63	0.63	0.62	0.64
7.0	0.50	0.50	0.52	0.52	0.53	0.53	0.55	0.55	0.56	0.57	0.57	0.58	0.59	0.59	0.60	0.59	0.60	0.61	0.62	0.62	0.61
7.5	0.50	0.50	0.51	0.52	0.53	0.54	0.55	0.55	0.55	0.56	0.57	0.58	0.58	0.58	0.59	0.60	0.60	0.60	0.62	0.61	0.61
8.0	0.50	0.50	0.51	0.52	0.52	0.53	0.54	0.54	0.55	0.56	0.56	0.57	0.57	0.57	0.58	0.58	0.59	0.59	0.59	0.60	0.61
8.5	0.50	0.50	0.51	0.52	0.52	0.53	0.53	0.54	0.55	0.55	0.56	0.57	0.56	0.57	0.58	0.58	0.58	0.59	0.59	0.60	0.59
9.0	0.50	0.50	0.51	0.51	0.52	0.52	0.53	0.53	0.54	0.54	0.55	0.56	0.56	0.57	0.57	0.58	0.58	0.59	0.58	0.59	0.59
9.5	0.50	0.50	0.51	0.51	0.52	0.52	0.53	0.54	0.54	0.55	0.55	0.56	0.56	0.56	0.57	0.57	0.57	0.57	0.58	0.58	0.60
10.0	0.50	0.50	0.51	0.51	0.52	0.53	0.53	0.53	0.53	0.54	0.55	0.55	0.56	0.56	0.56	0.56	0.56	0.57	0.58	0.58	0.58
10.5	0.50	0.50	0.51	0.51	0.52	0.52	0.53	0.53	0.53	0.54	0.54	0.54	0.55	0.55	0.56	0.56	0.57	0.57	0.57	0.57	0.58
11.0	0.50	0.50	0.51	0.51	0.51	0.52	0.52	0.52	0.53	0.53	0.54	0.55	0.55	0.55	0.56	0.56	0.56	0.56	0.57	0.58	0.57

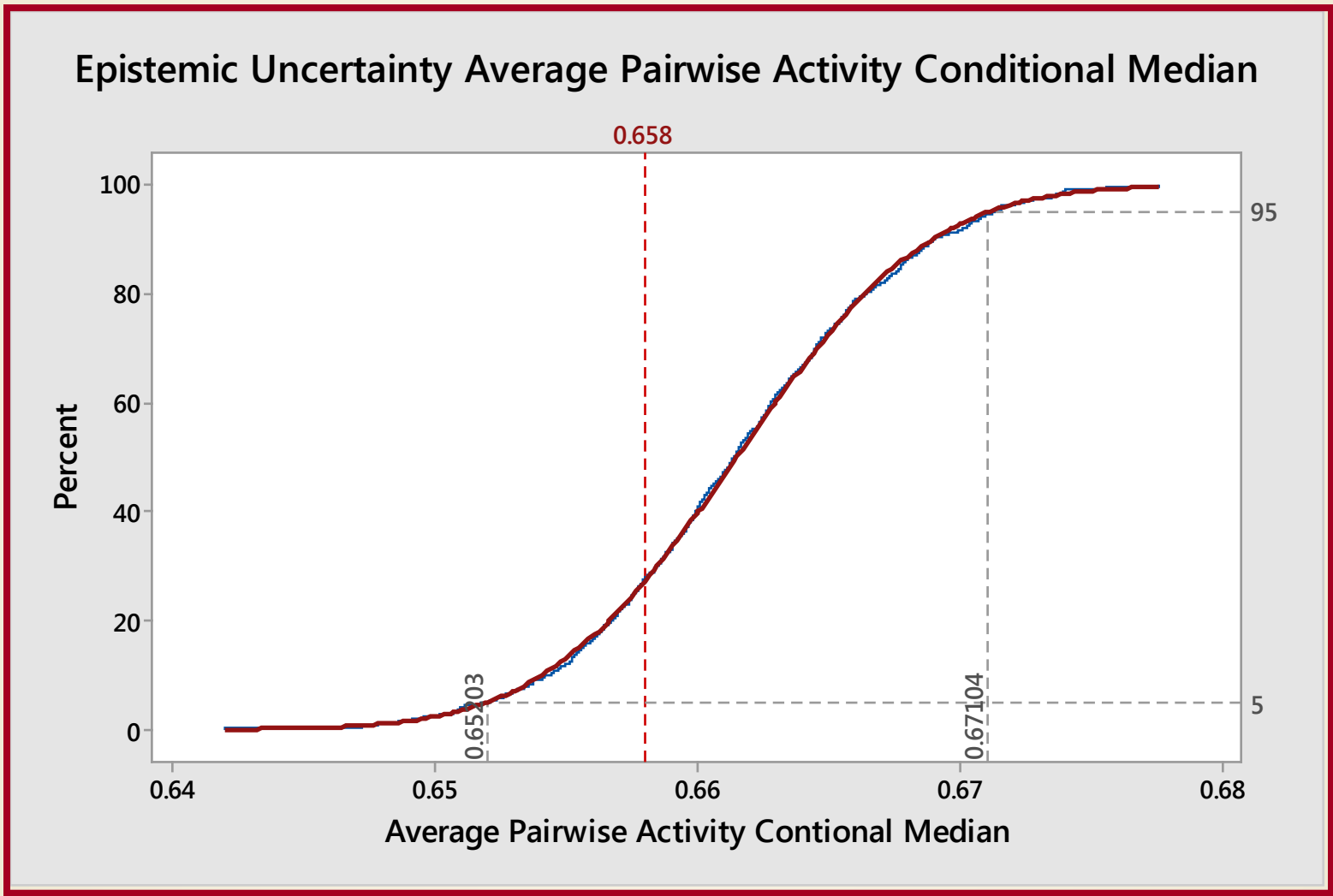
Set  $n = 3.5$  and  $\beta = 13$

# STEP 3: Evaluate Uncertainty given selected $\beta$ and $n$



Setting  $n = 3.5$  and  $\beta = 13$

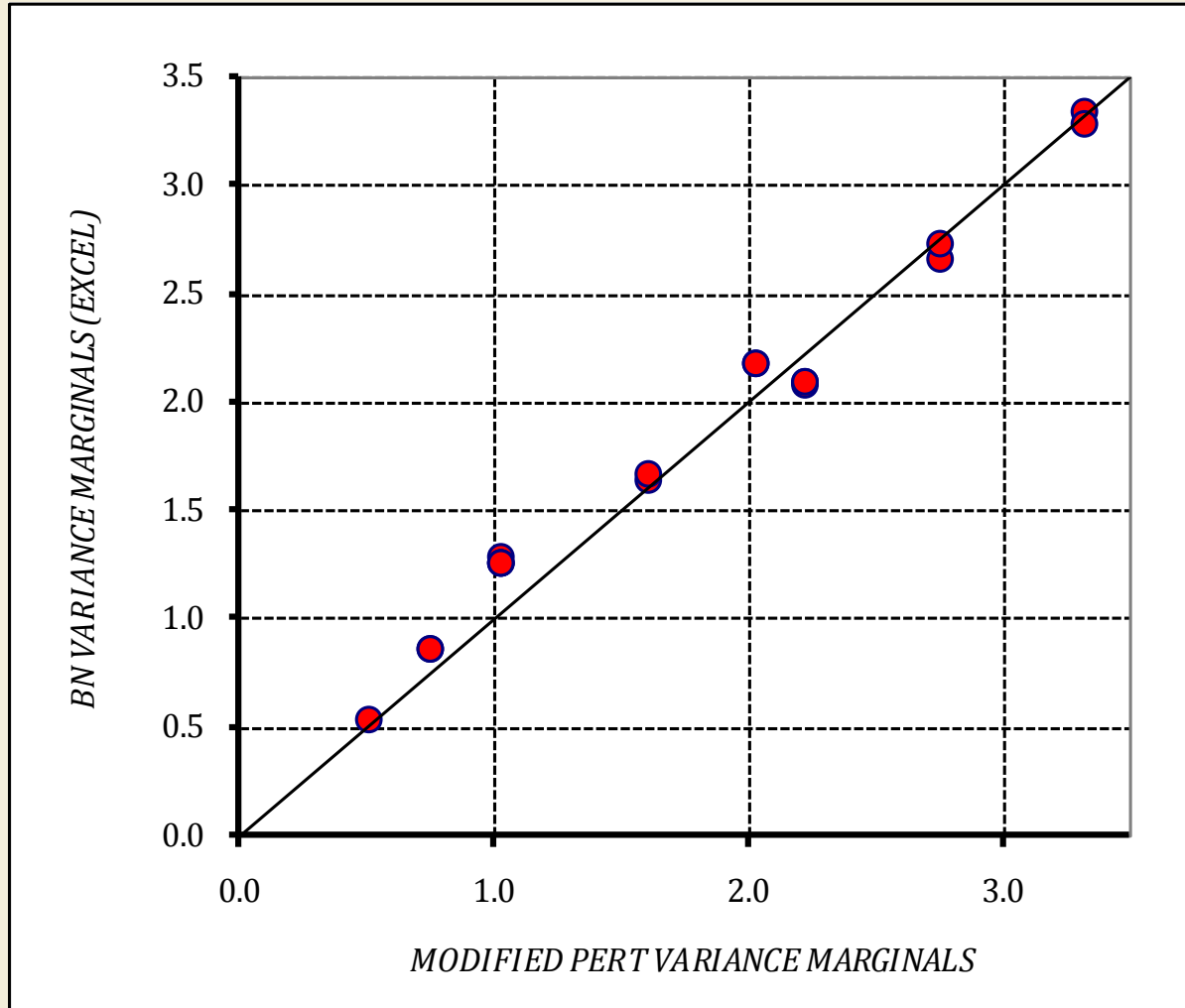
# STEP 3: Evaluate Uncertainty given selected $\beta$ and $n$



Setting  $n = 3.5$  and  $\beta = 13$

# Additional Feedback for selected values of $\beta$ and $n$

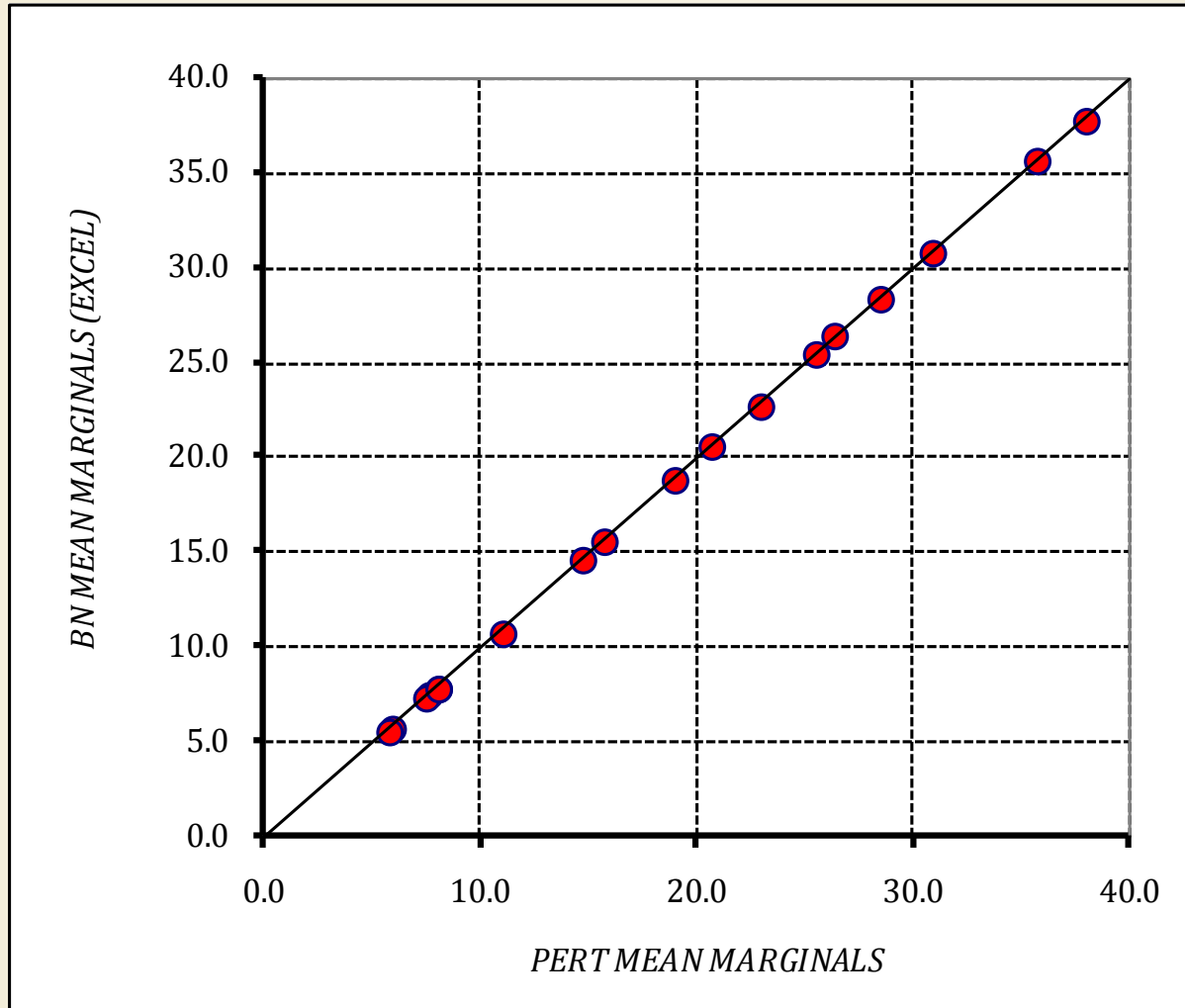
Setting  $n = 3.5$  and  $\beta = 13$



Activity Variances are close to Modified PERT Variances

# Additional Feedback for selected values of $\beta$ and $n$

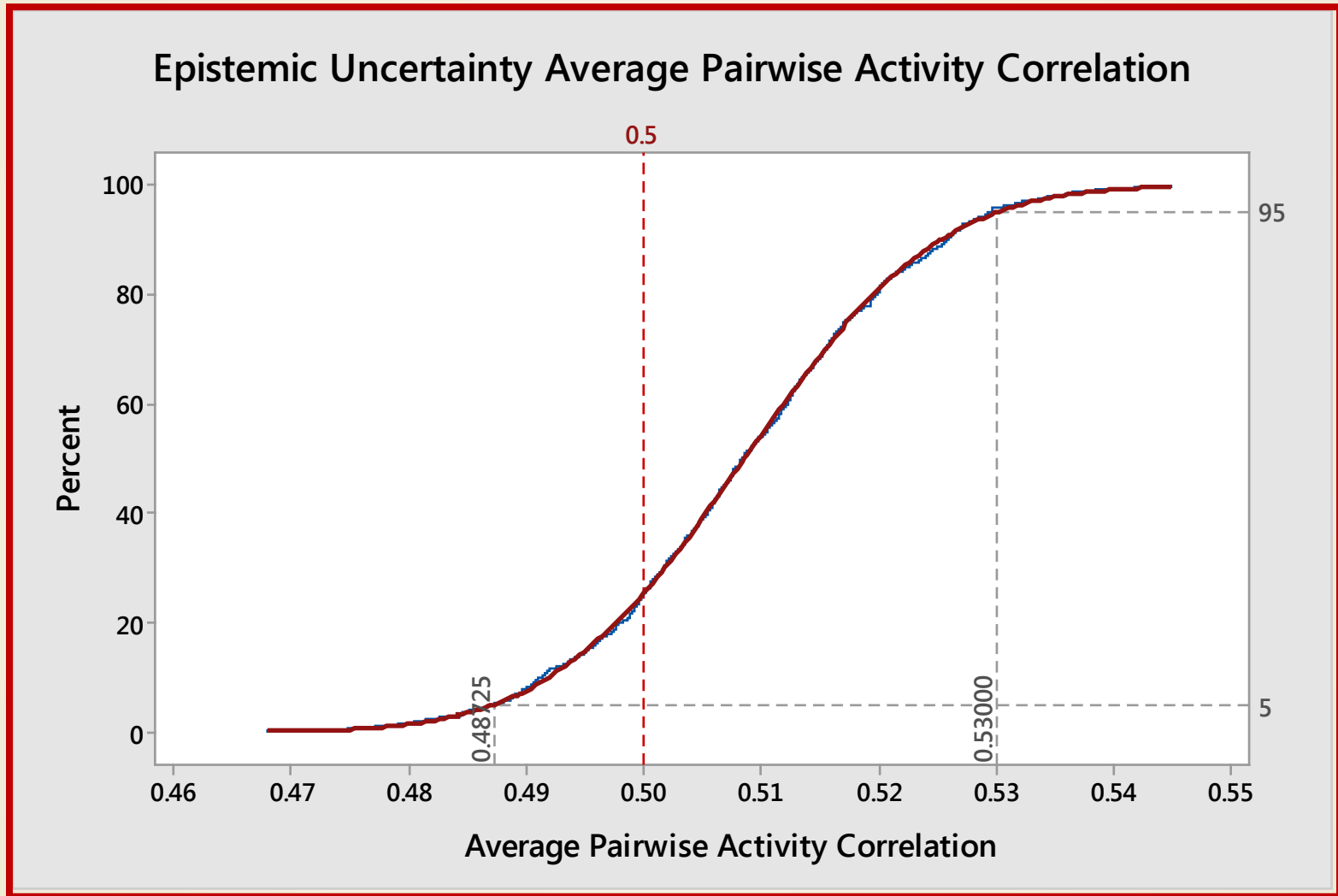
Setting  $n = 3.5$  and  $\beta = 13$



Activity Means are close to PERT Means

# Additional Feedback for selected values of $\beta$ and $n$

Setting  $n = 3.5$  and  $\beta = 13$



Average Pairwise Activity Correlation is close to 0.5



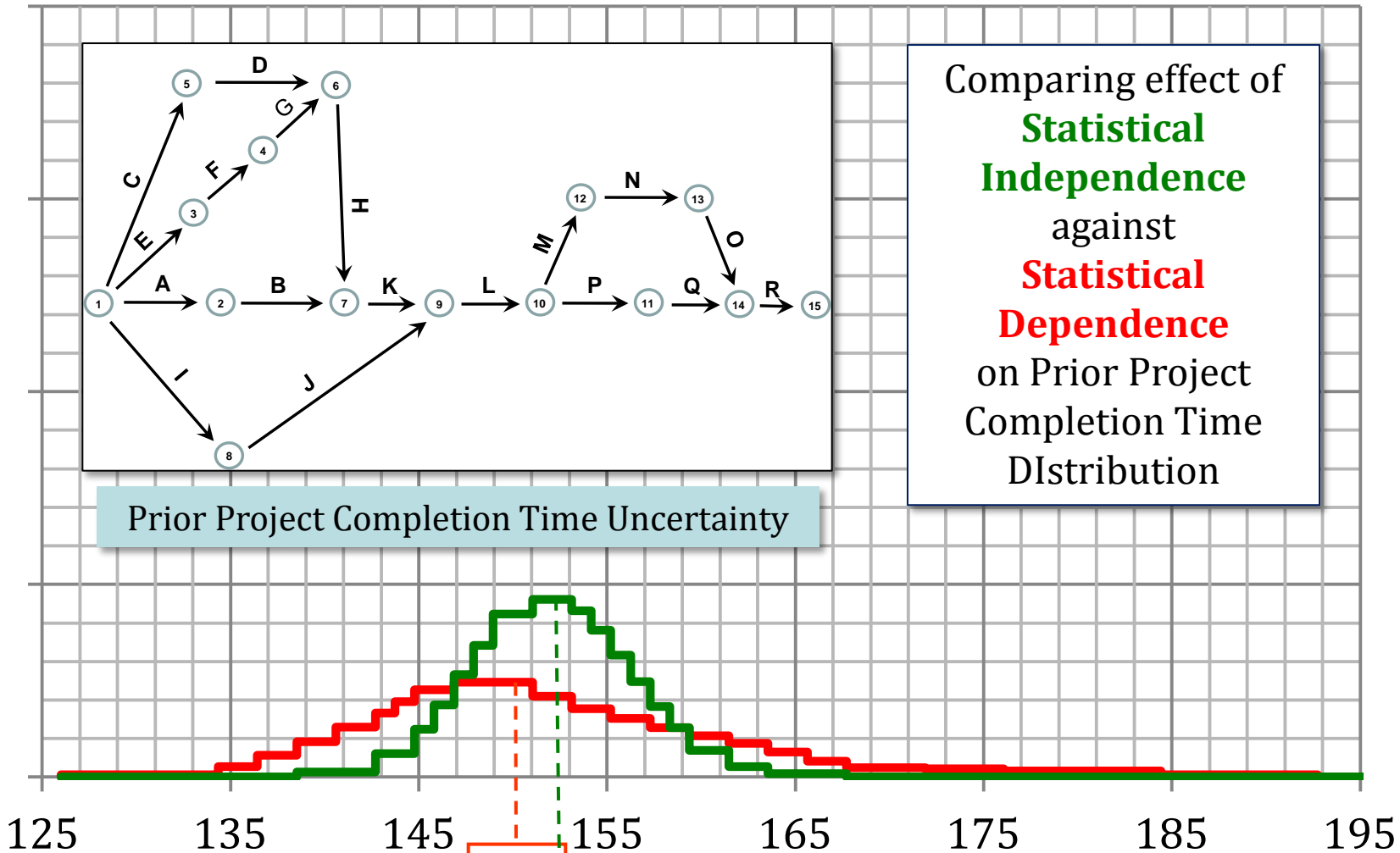
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Cumulative Probability

40%  
30%  
20%  
10%  
0%



Prior Project Completion Time Uncertainty

125 135 145 155 165 175 185 195

150.8  
152.2

CRITICAL PATH: E F G H K L P Q R

Project Completion Time

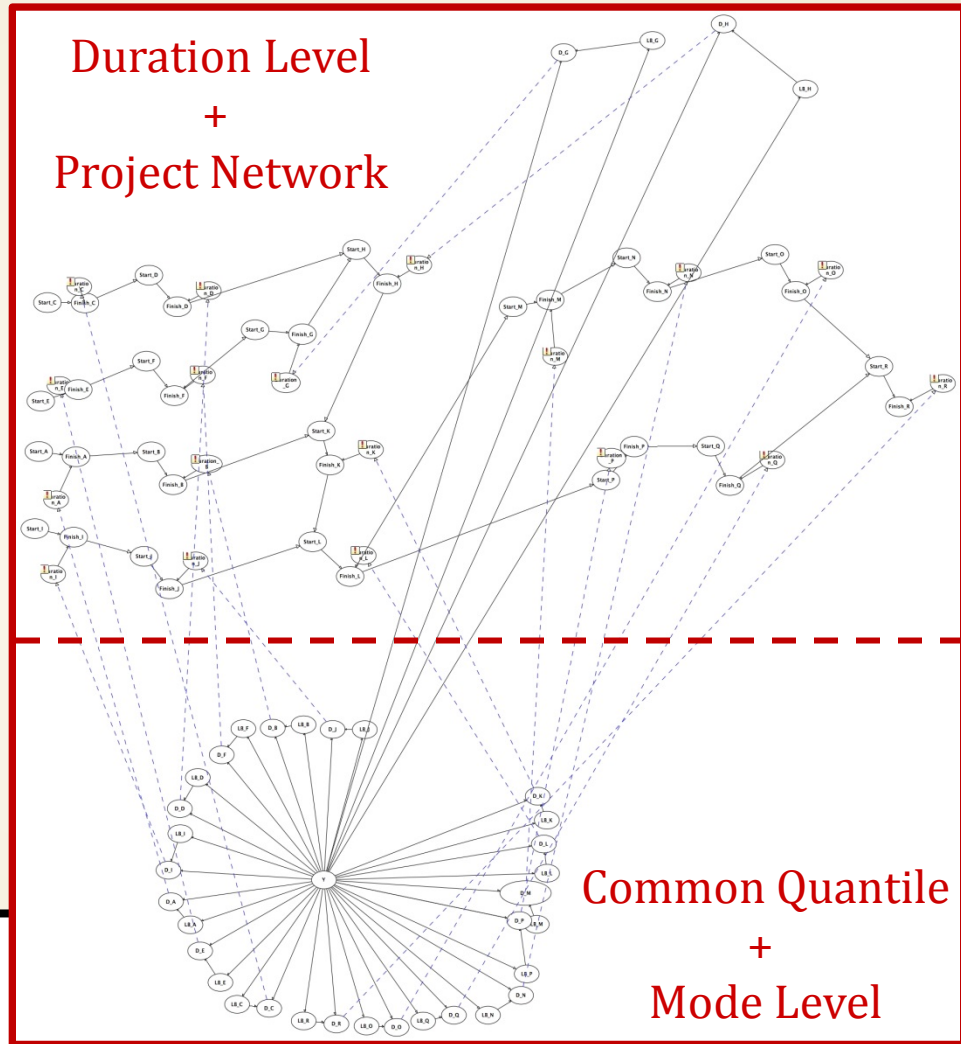
— DEP P — IND P

# Bayesian Network Dependence Model for Project Risk Analysis (BNDM\_PRA)

Project and Bayes Network Representation

in Software Agena Risk

Posterior Analysis performed in Software:



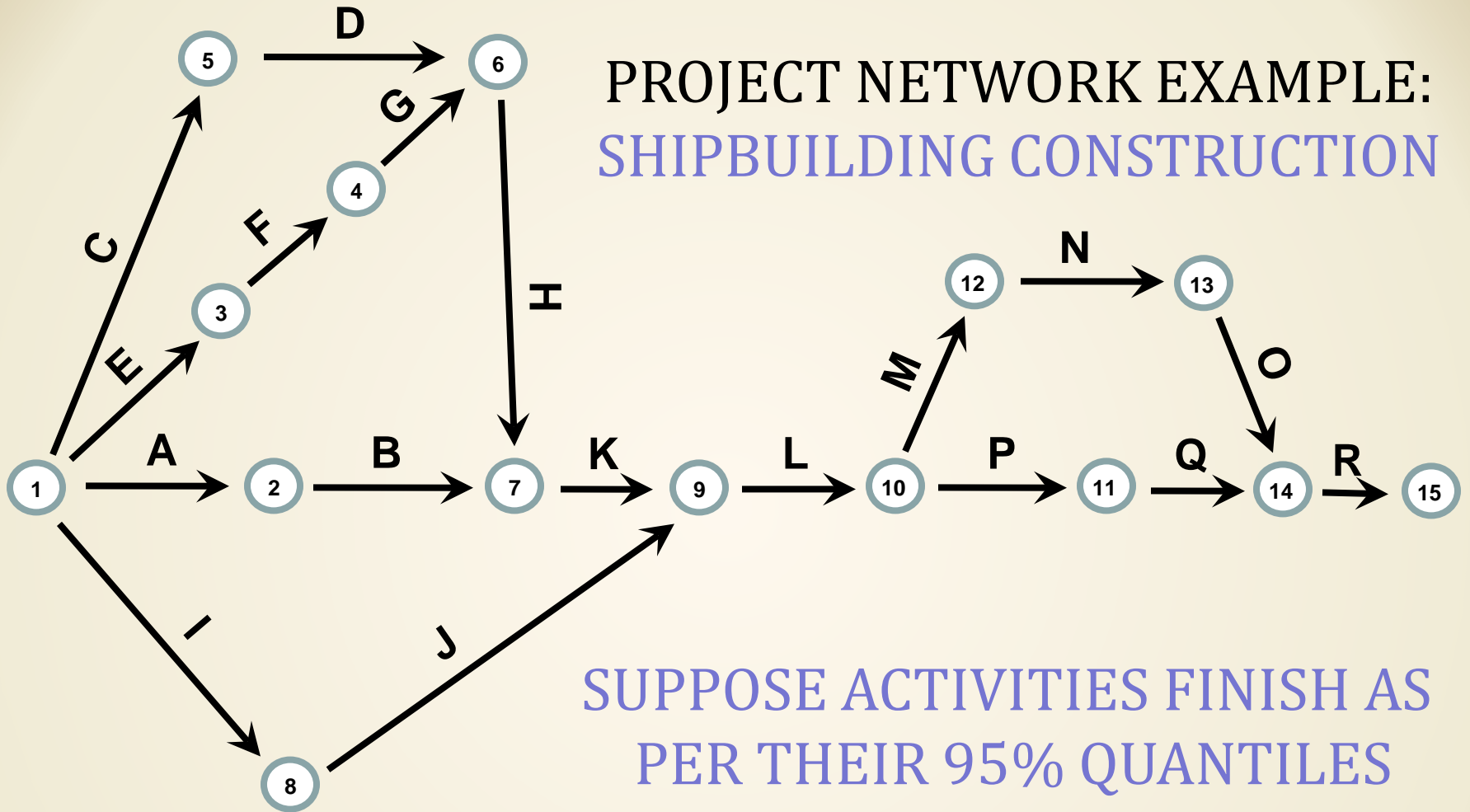
Bayesian Network and Simulation Software for Risk Analysis and Decision Support

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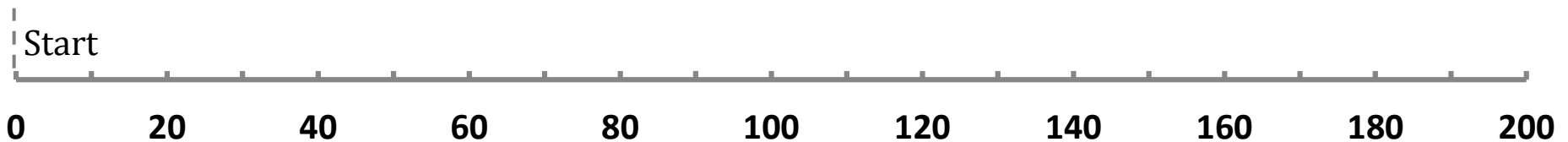
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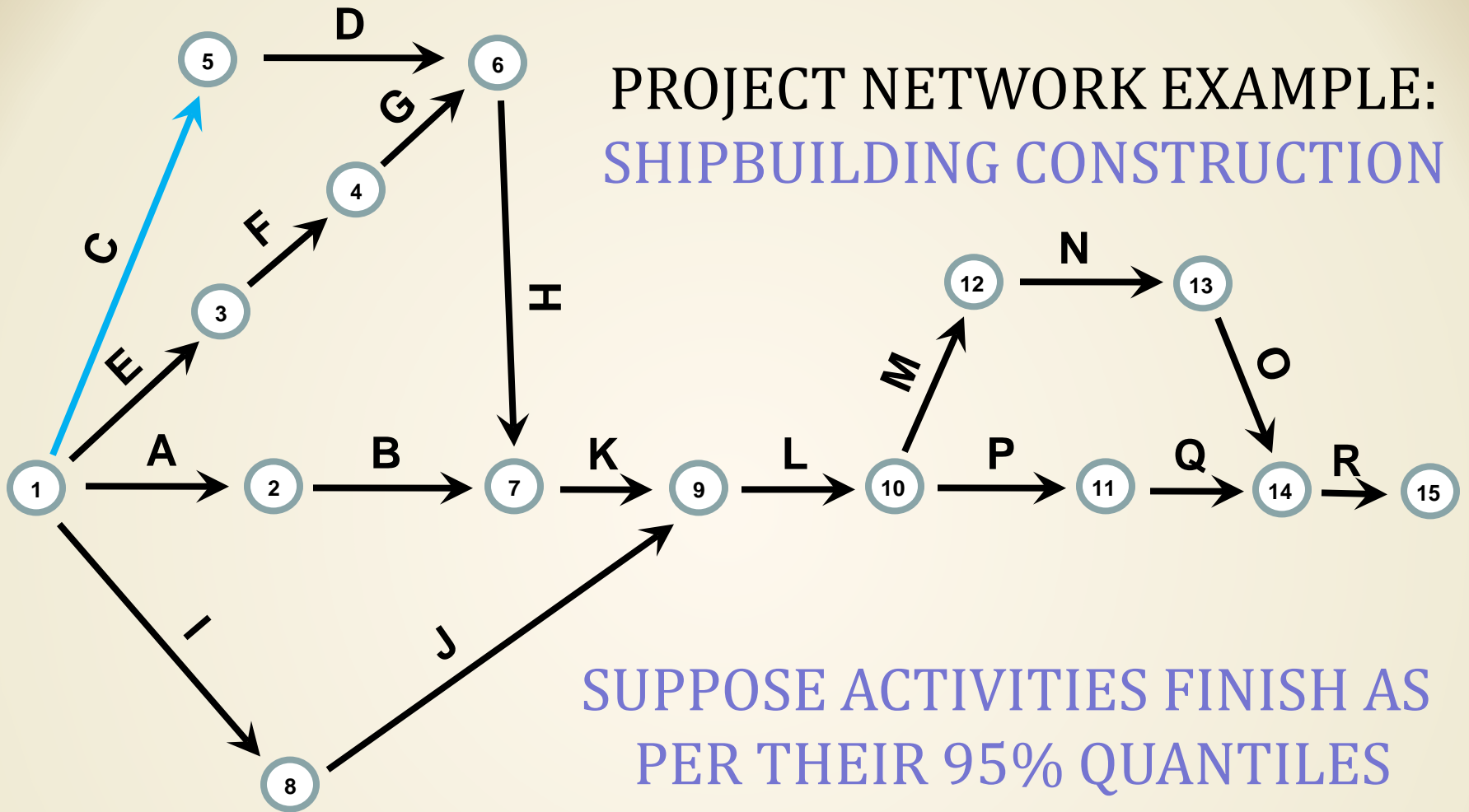
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



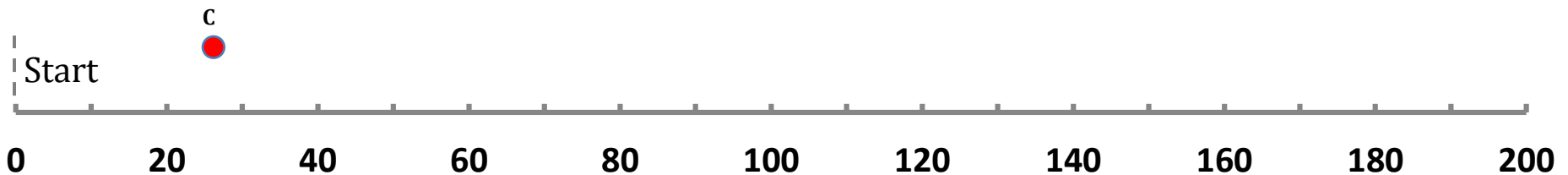
SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



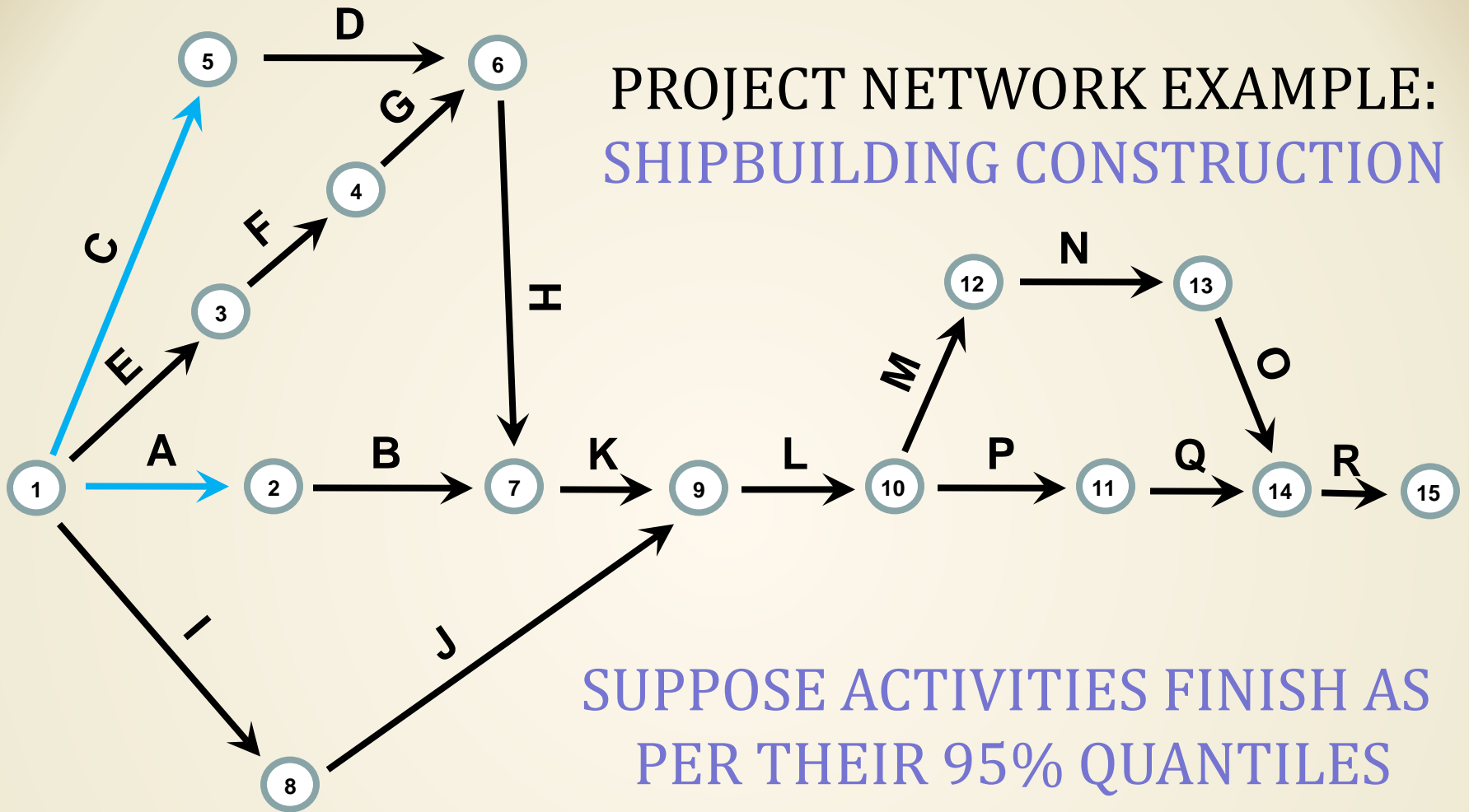
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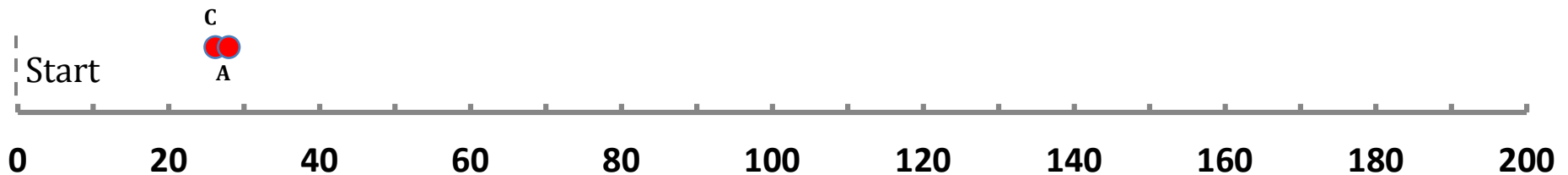
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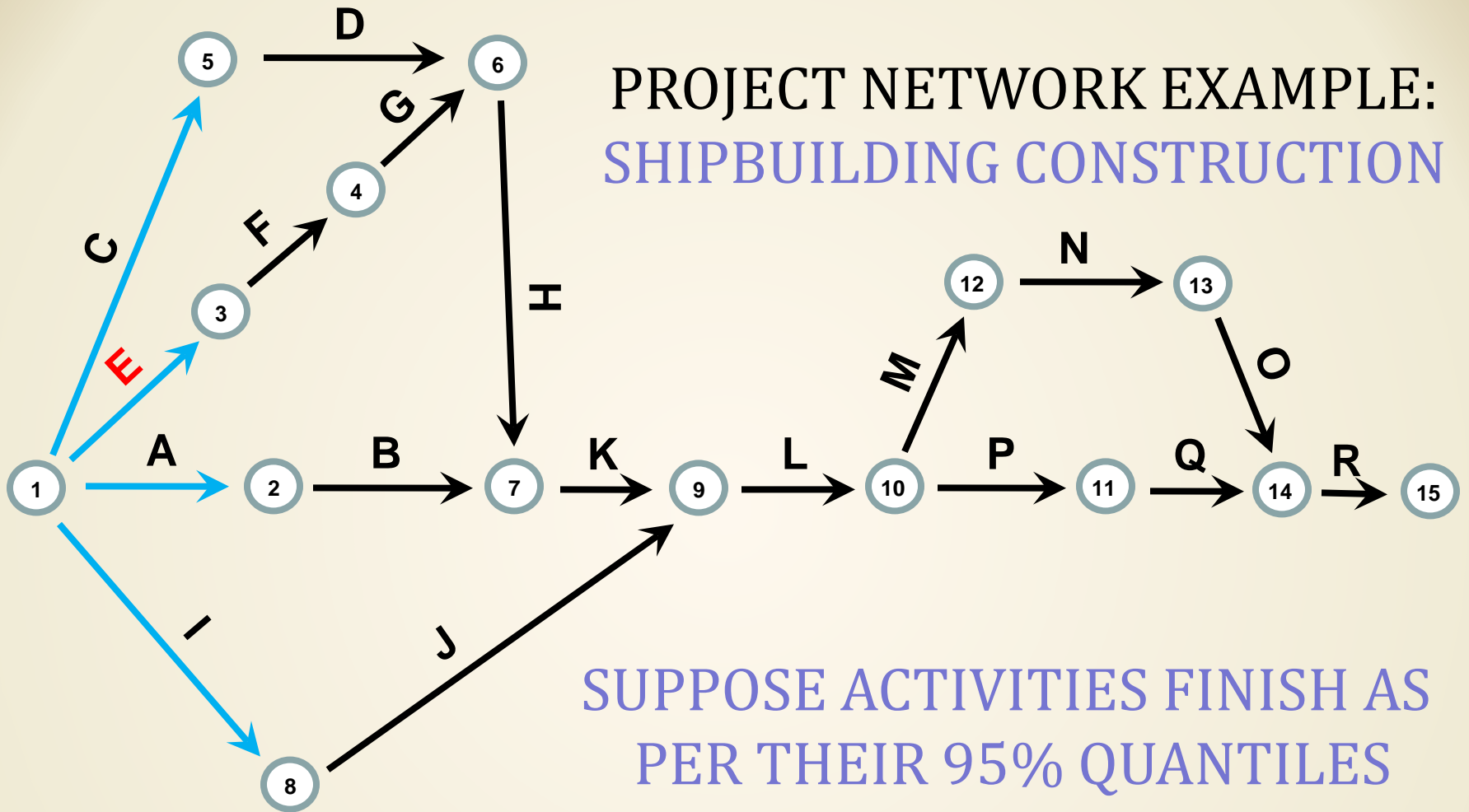
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# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



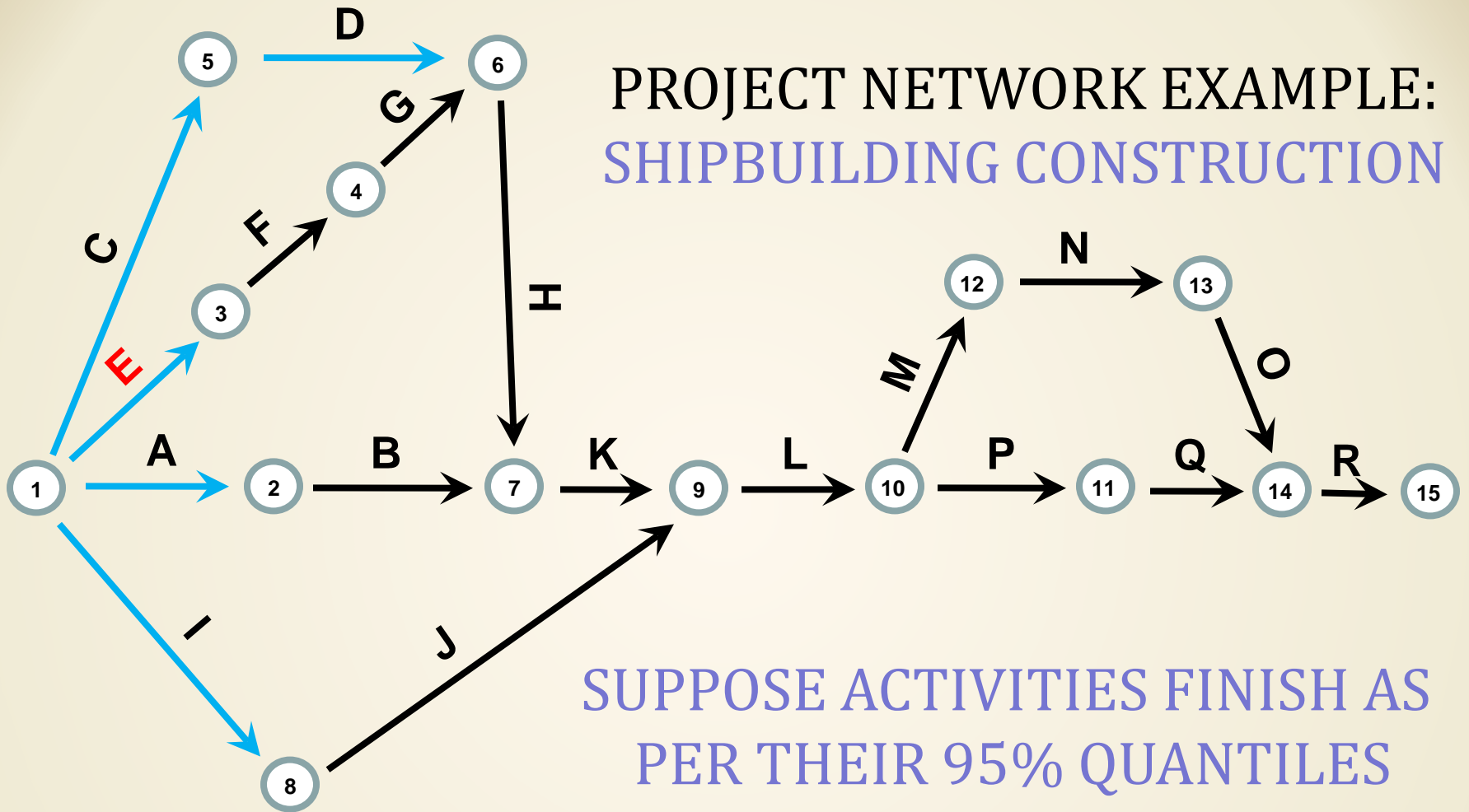
SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES

C E  
A I

Start

0 20 40 60 80 100 120 140 160 180 200

# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



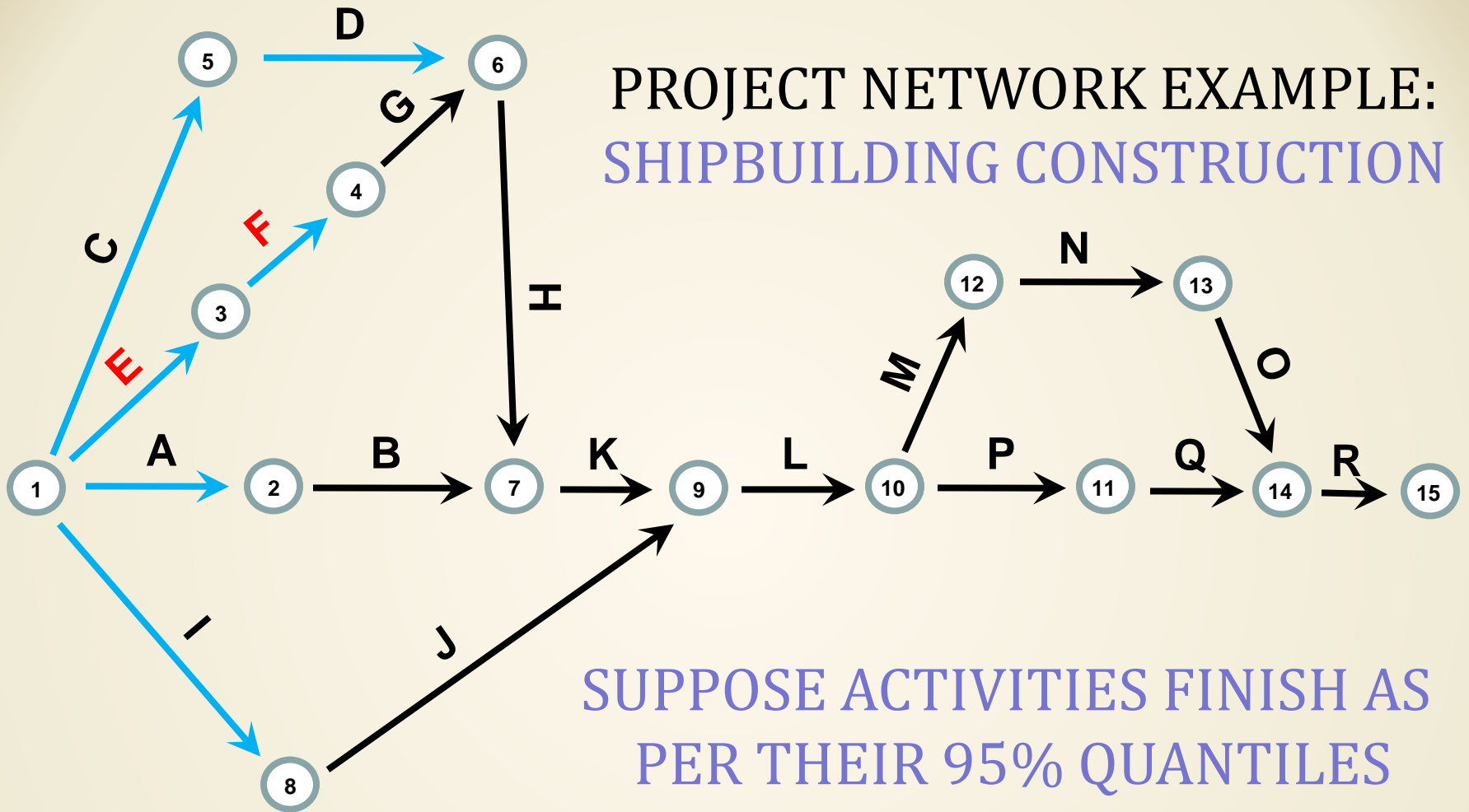
SUPPOSE ACTIVITIES FINISH AS  
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C E D  
A I

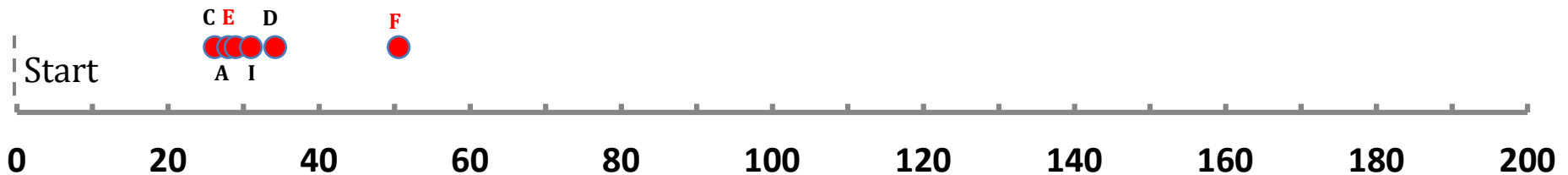
Start

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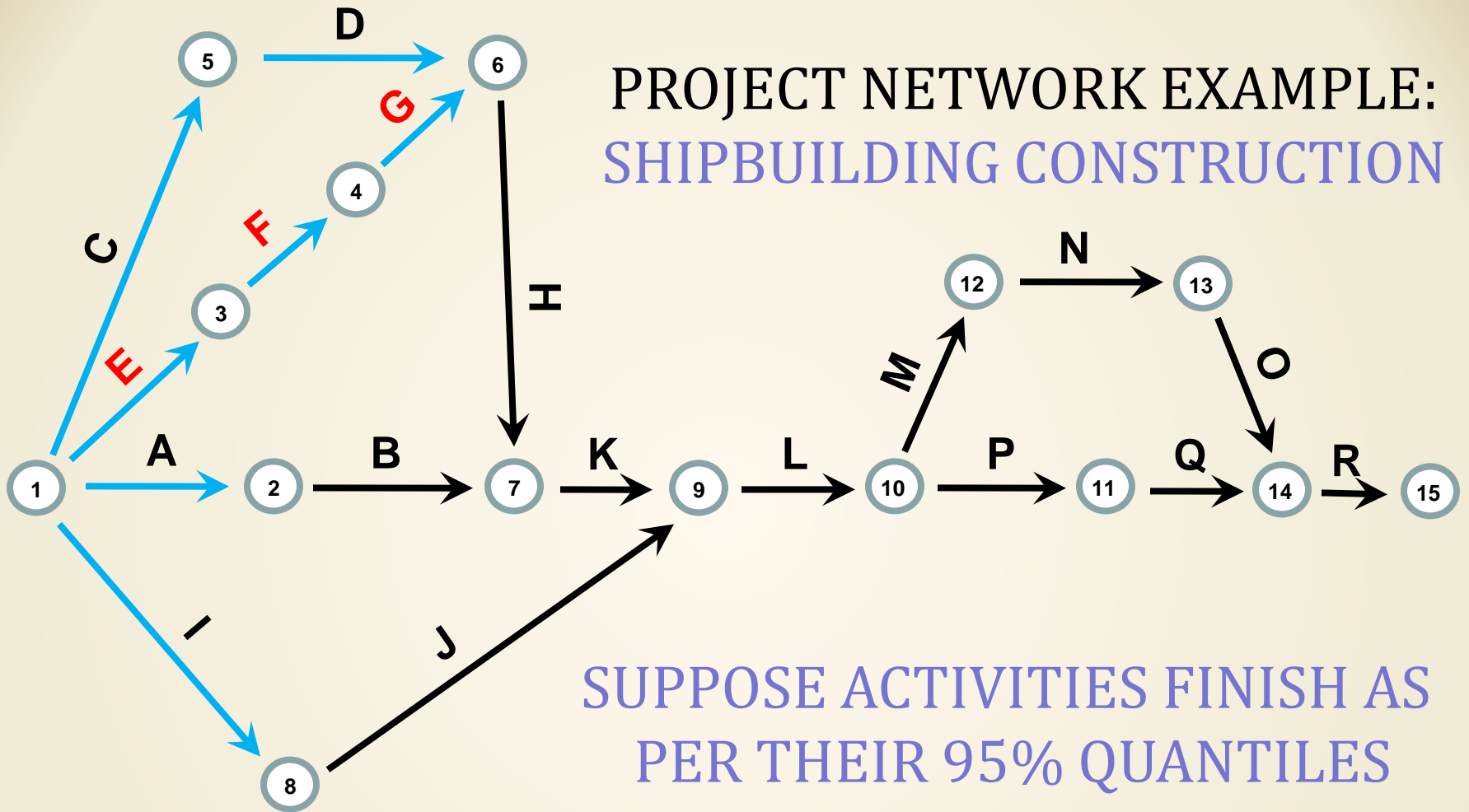
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



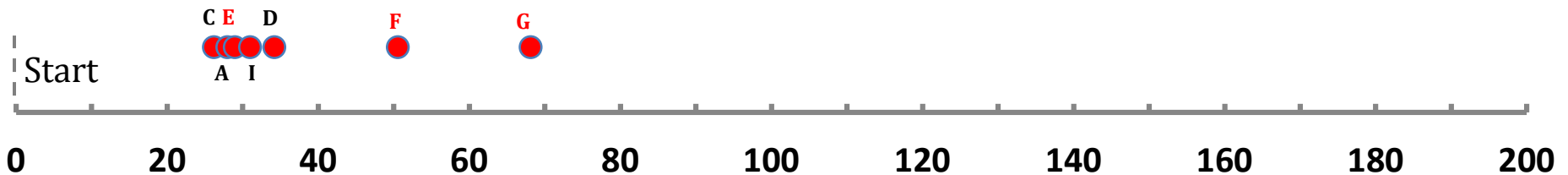
SUPPOSE ACTIVITIES FINISH AS  
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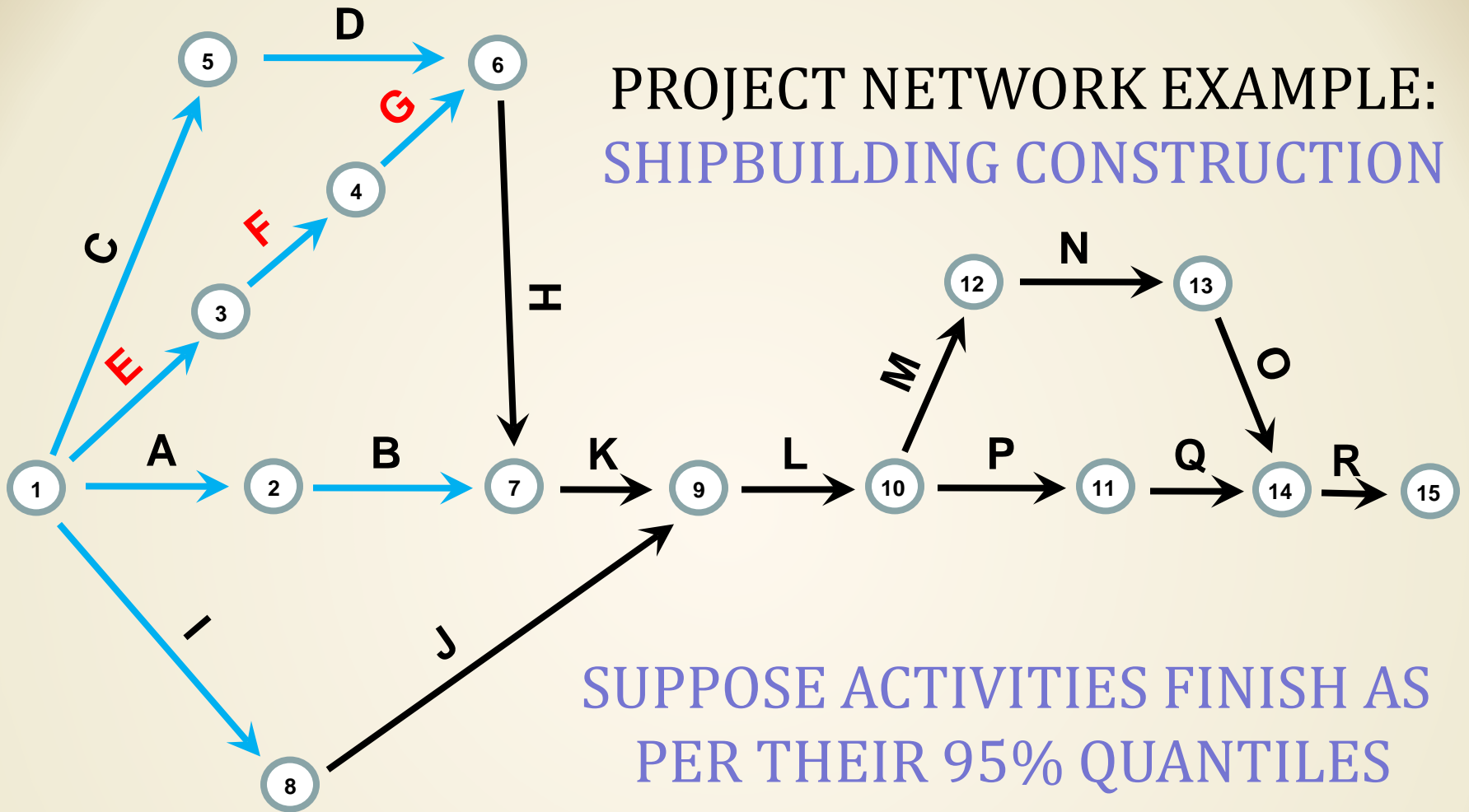
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



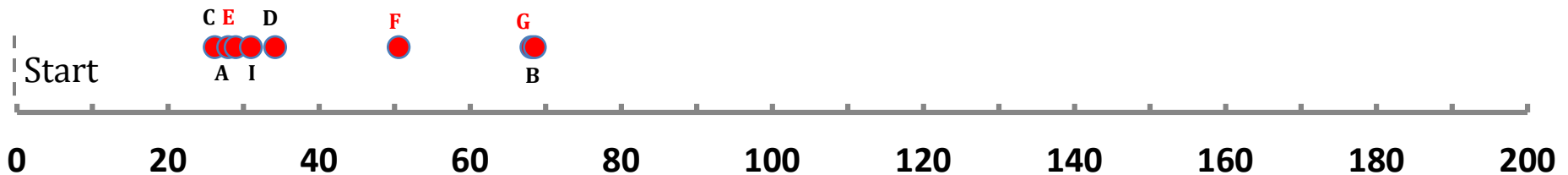
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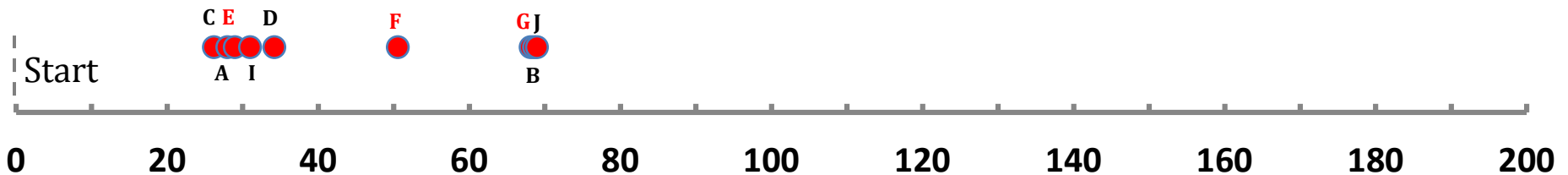
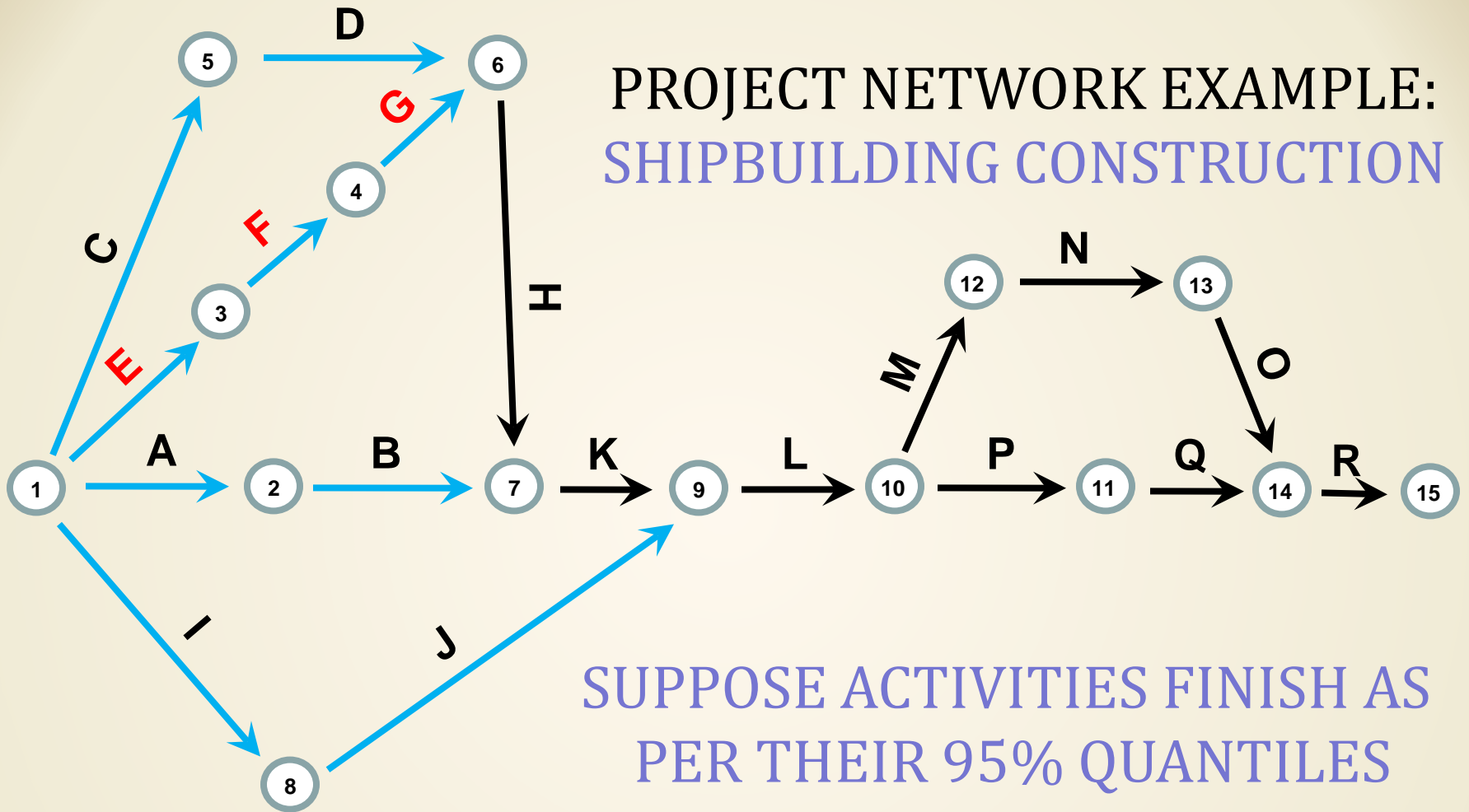
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



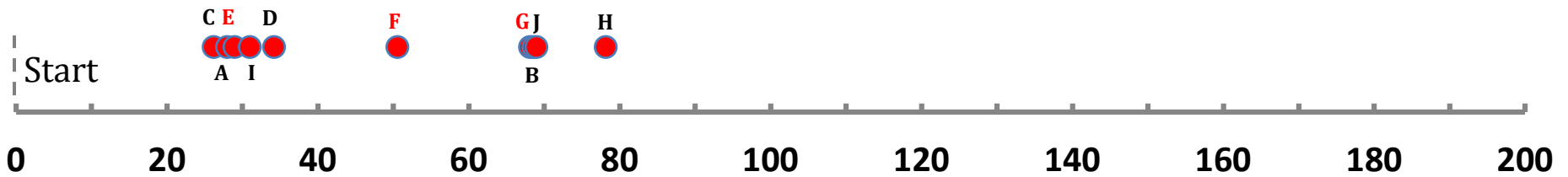
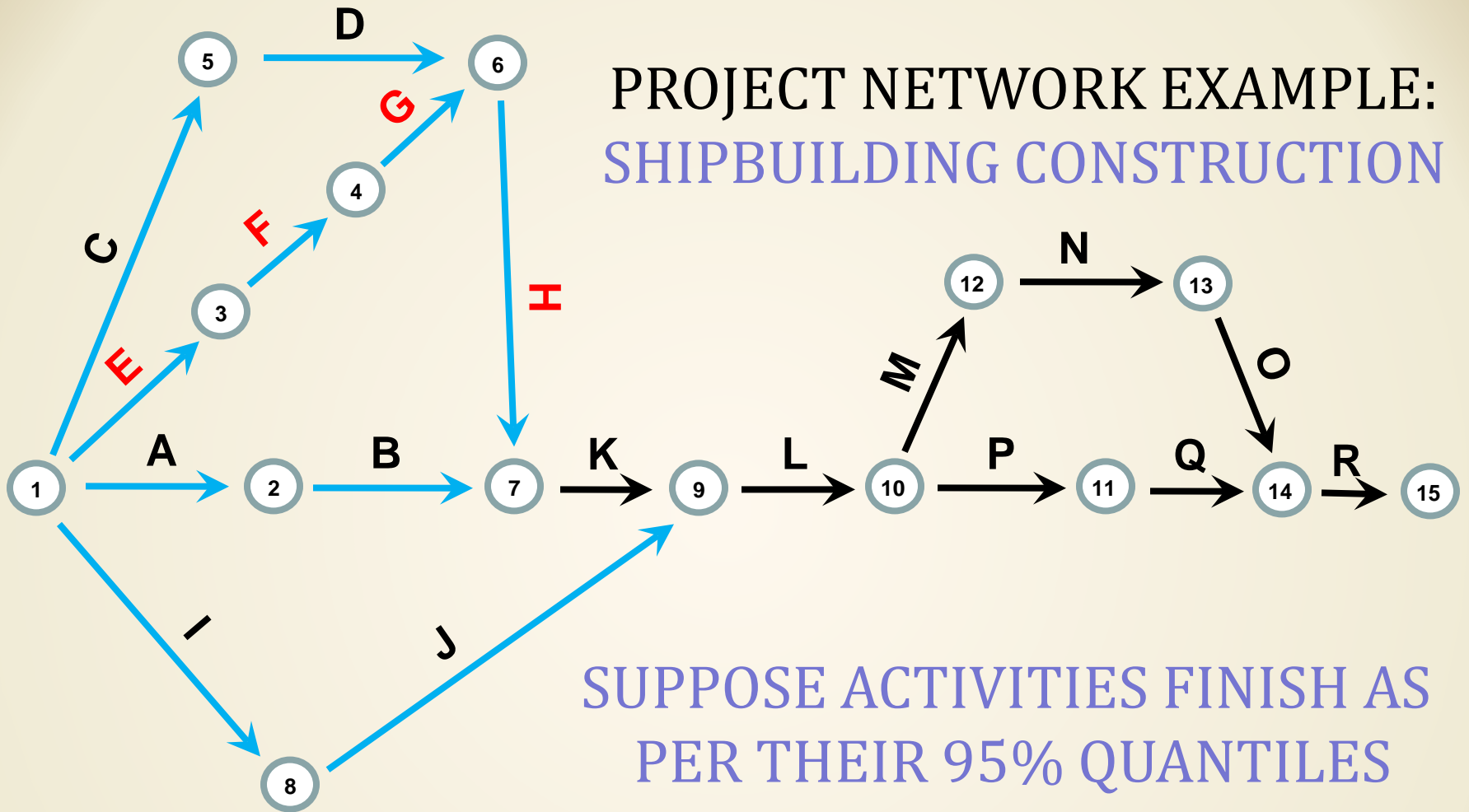
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



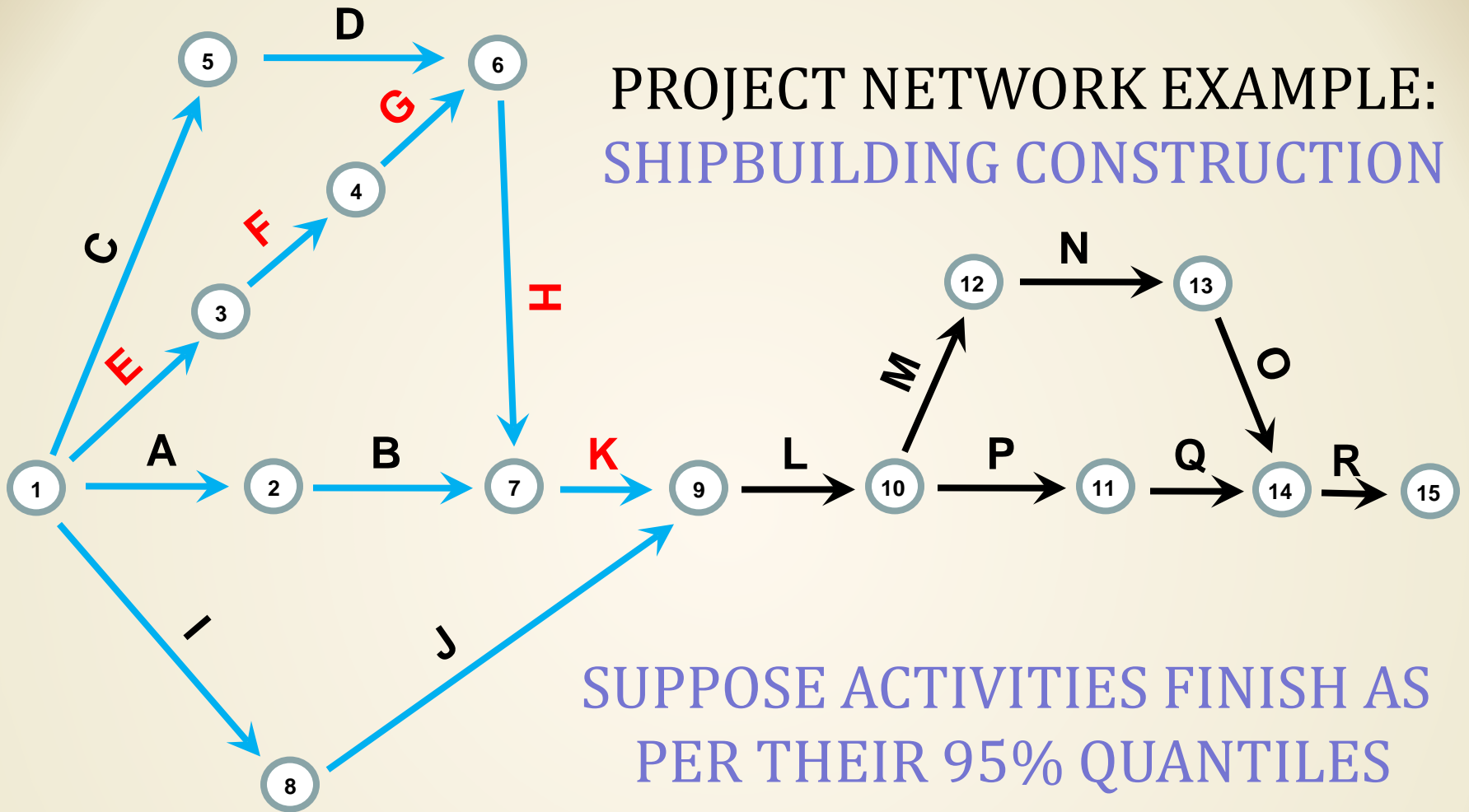


# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION

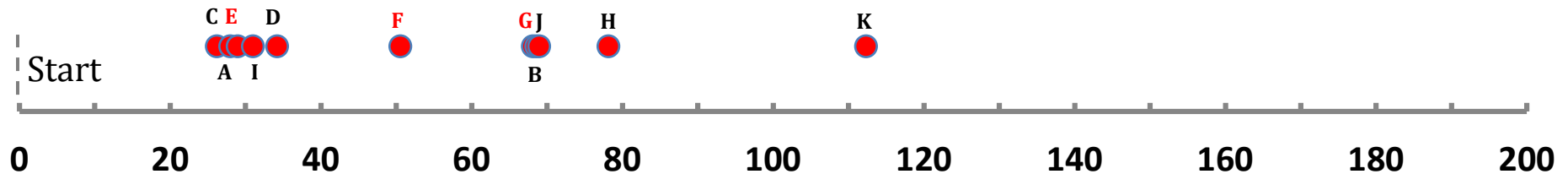
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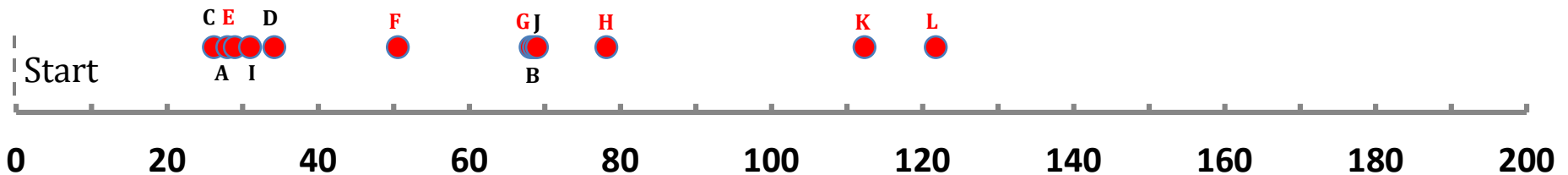
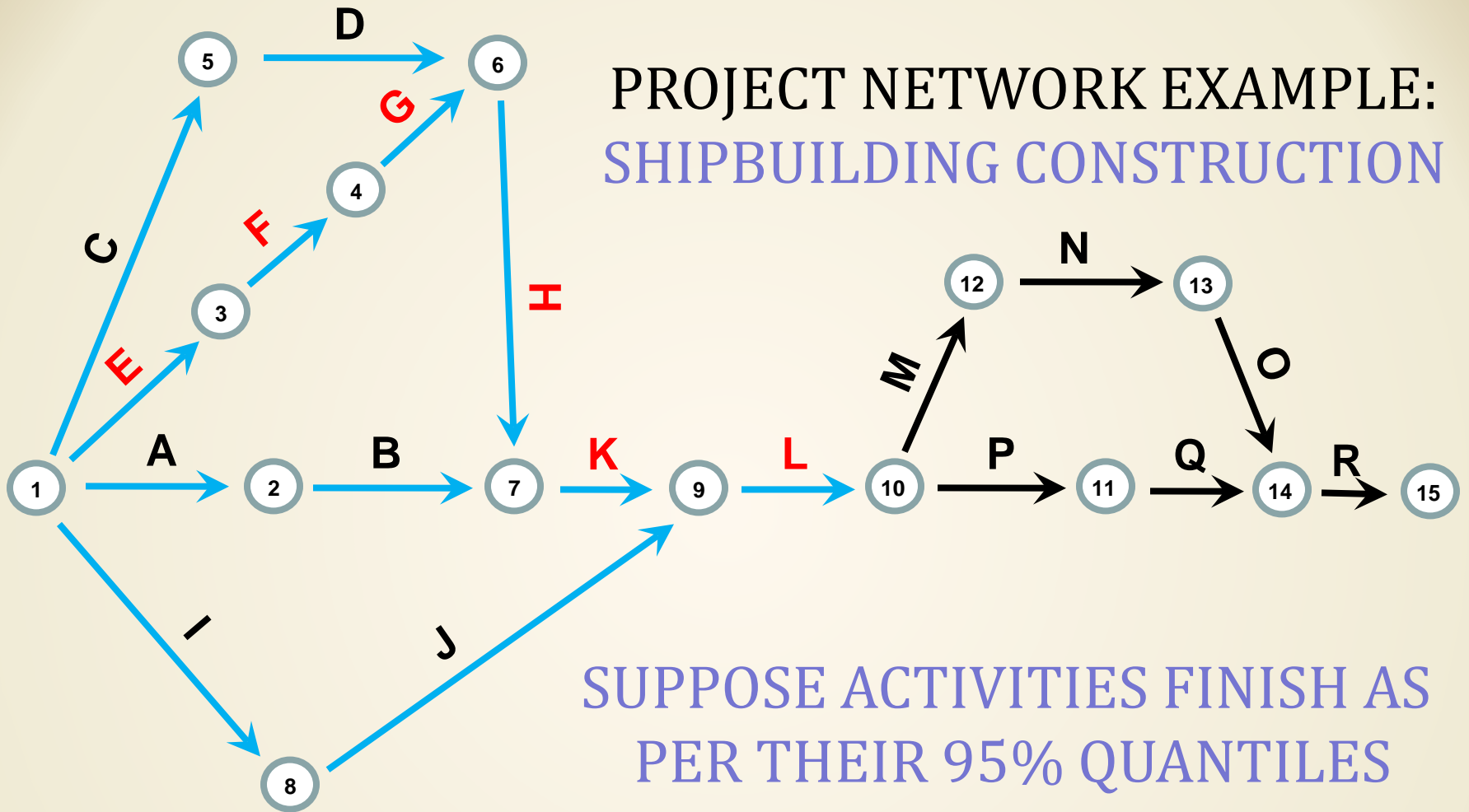
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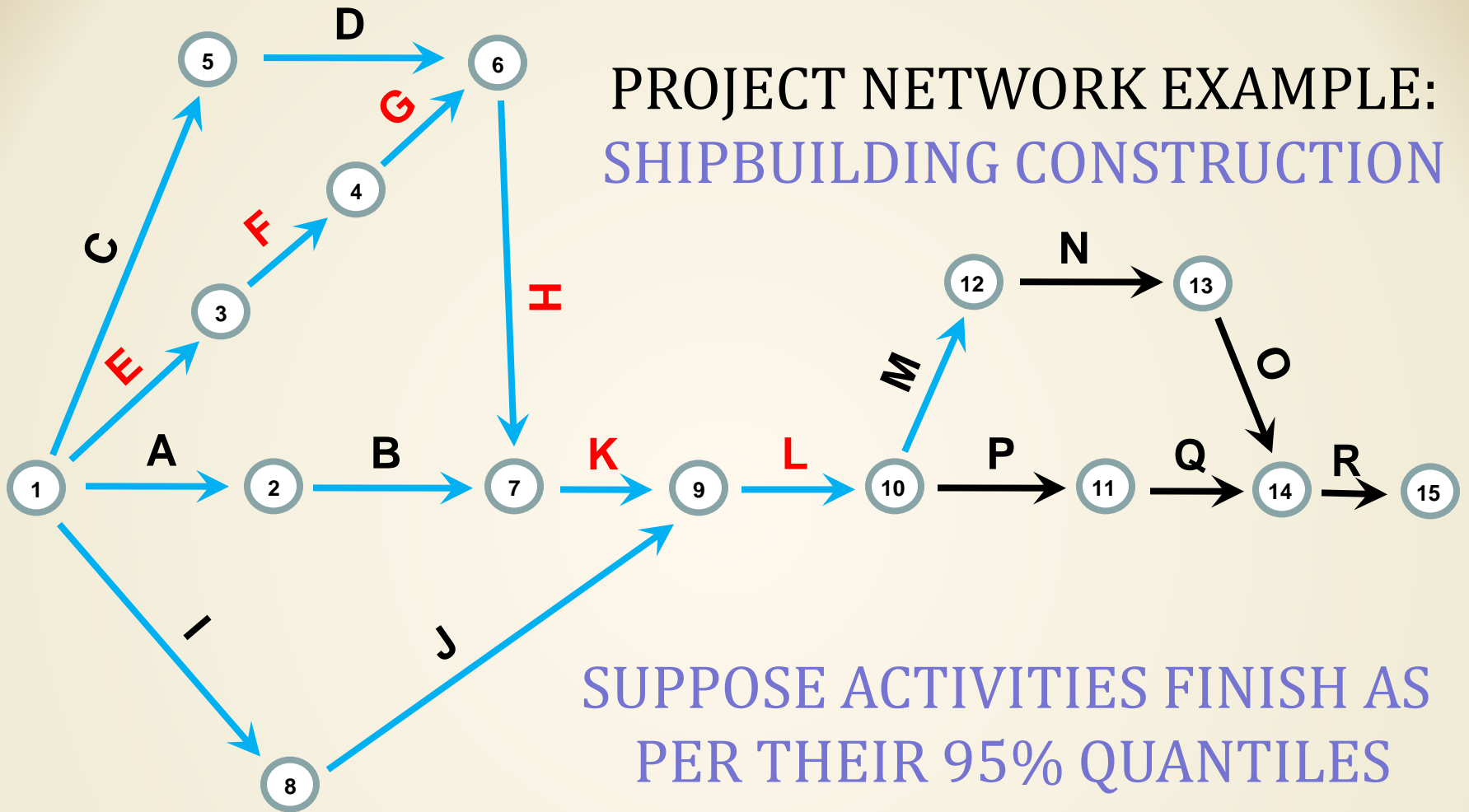
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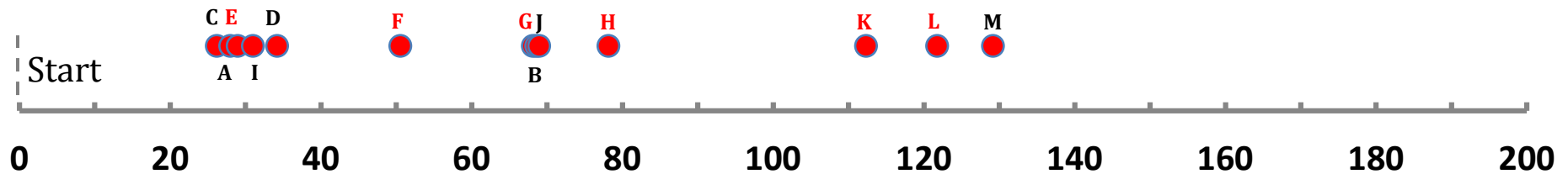
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



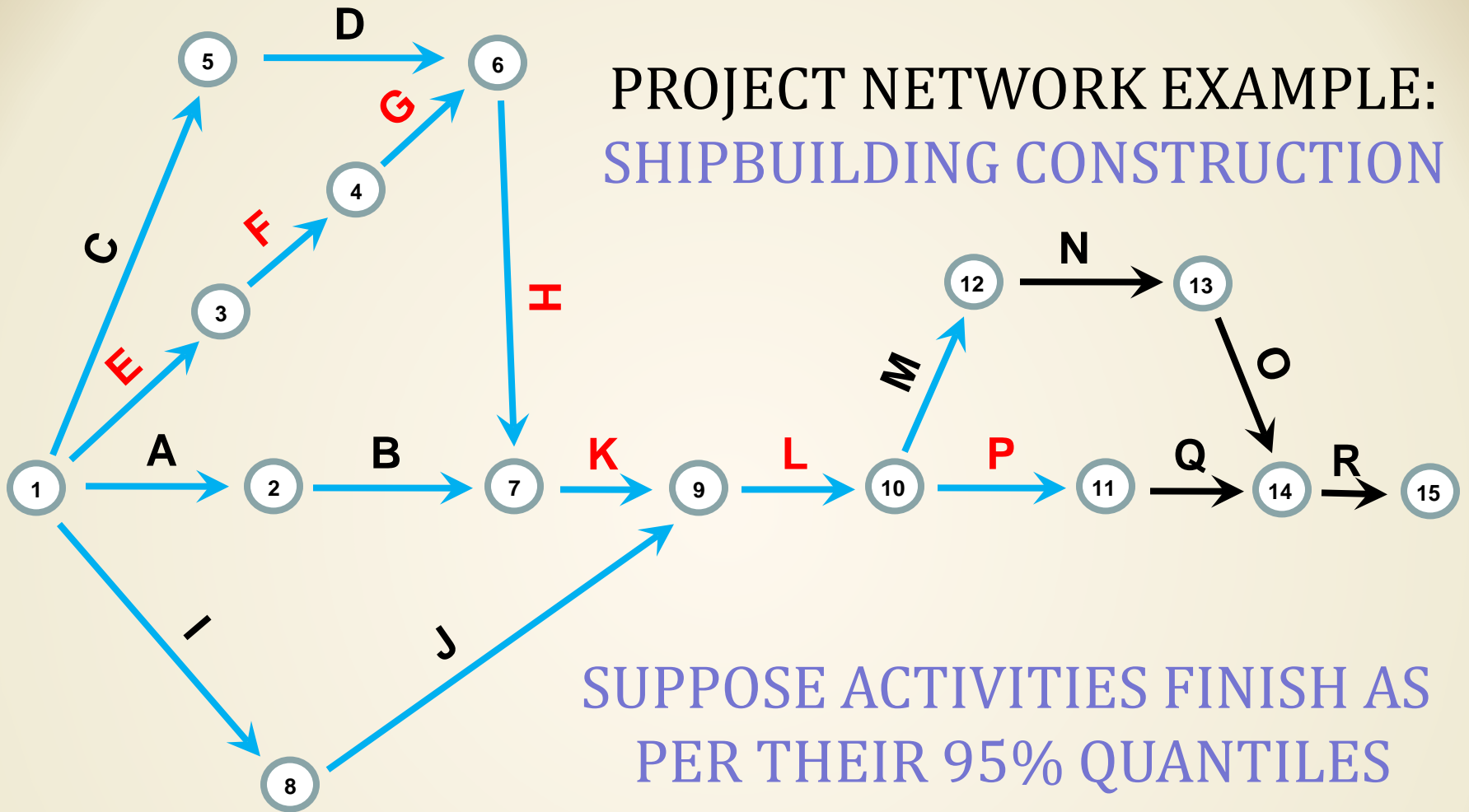
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



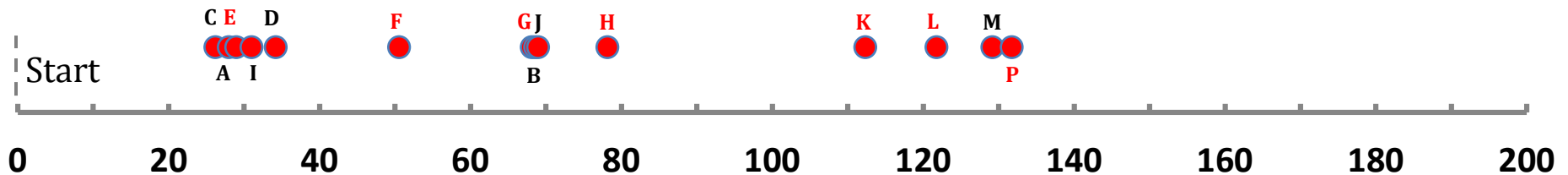
SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



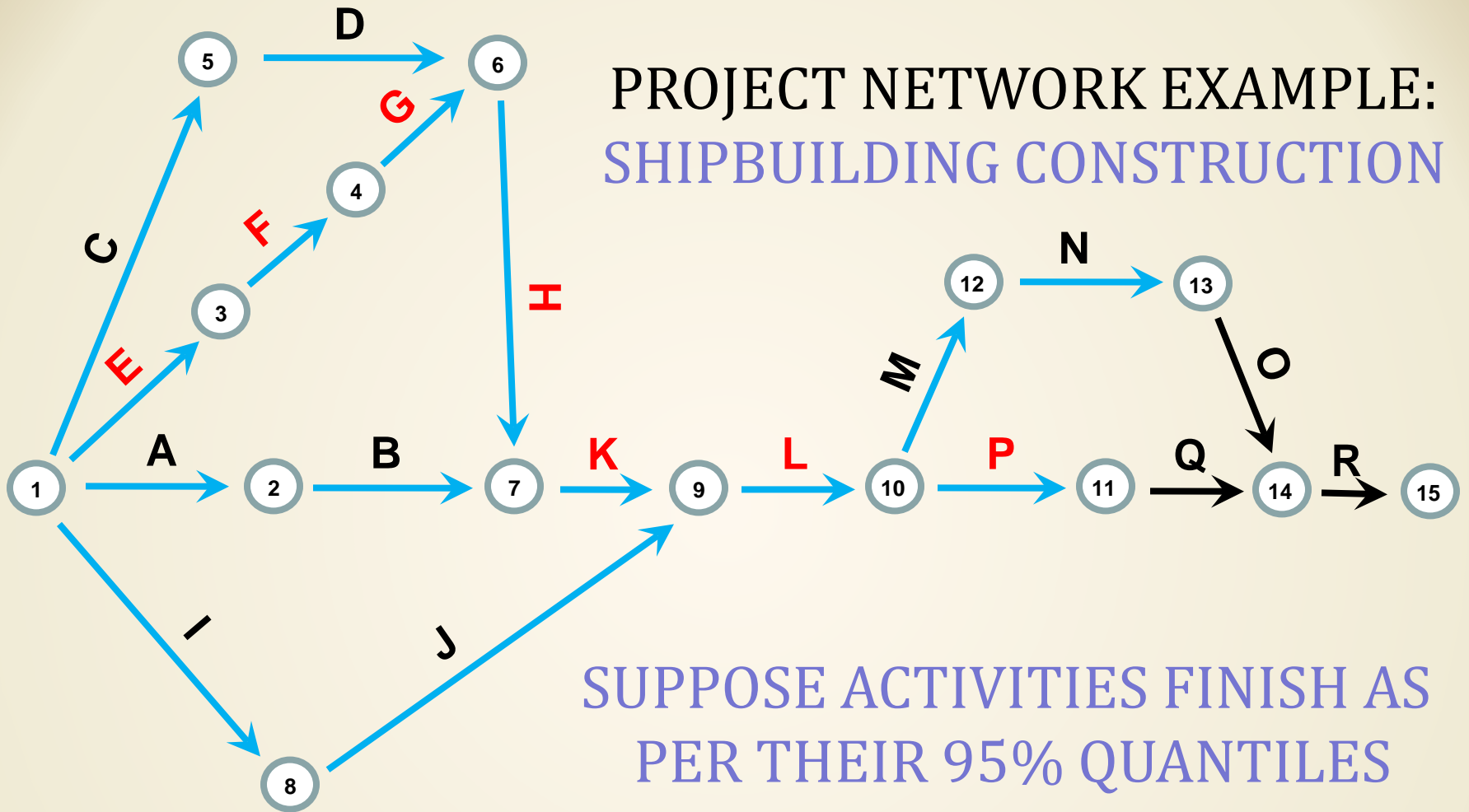
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



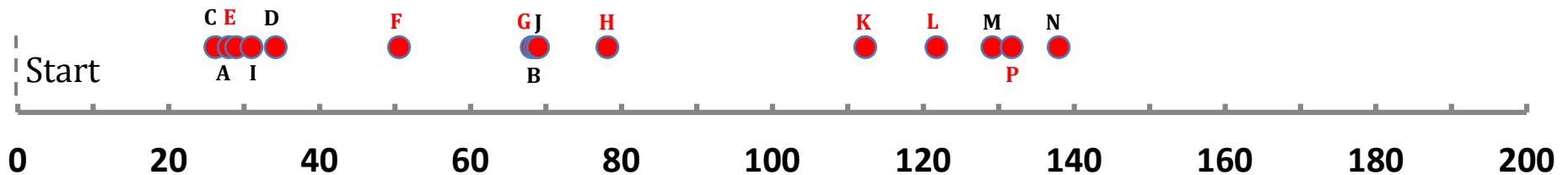
SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



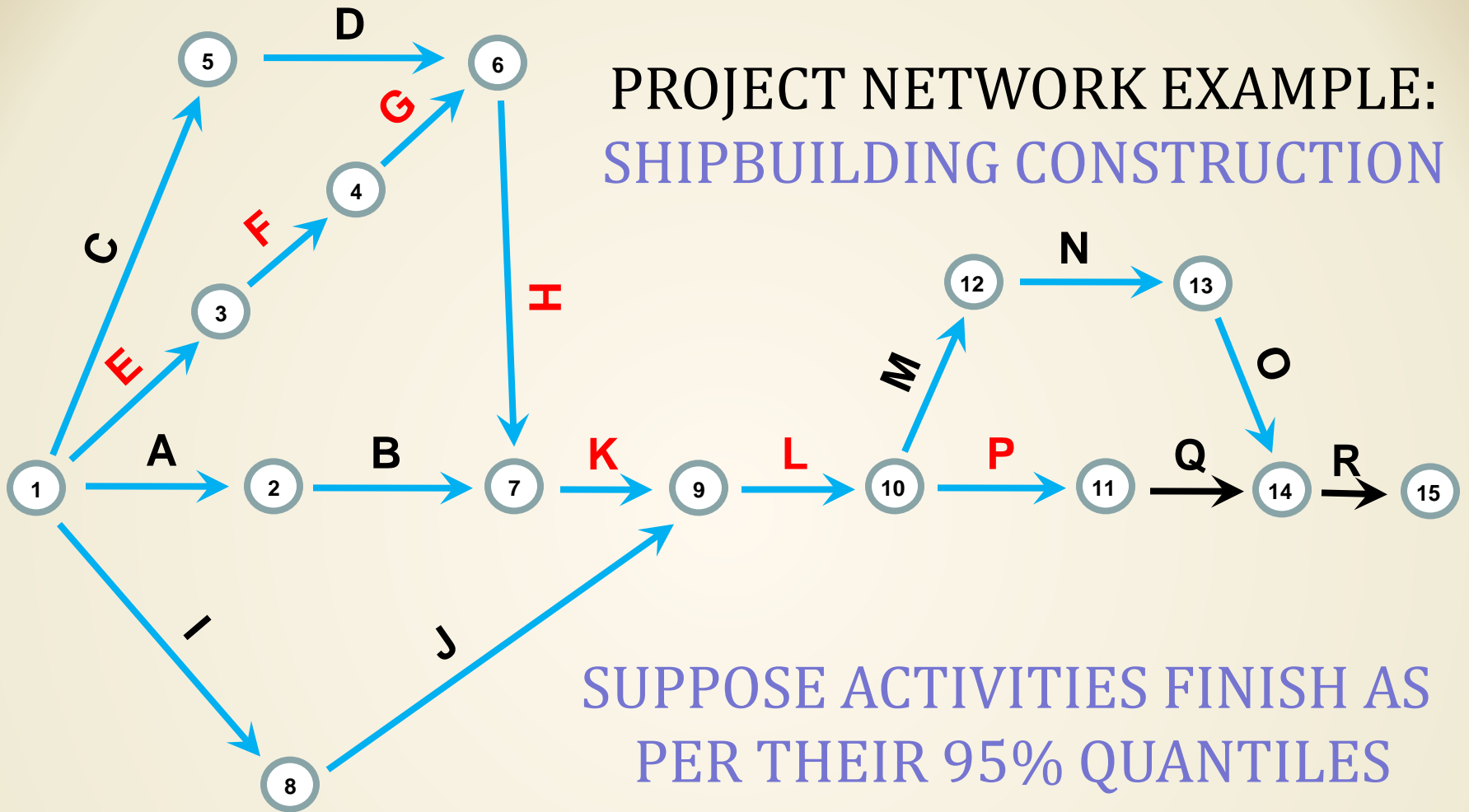
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



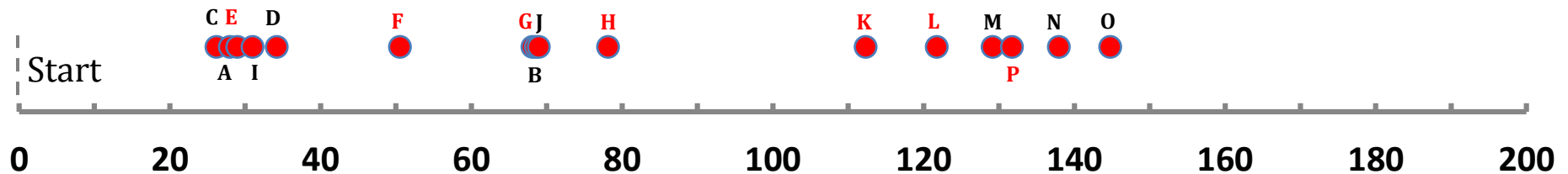
SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION

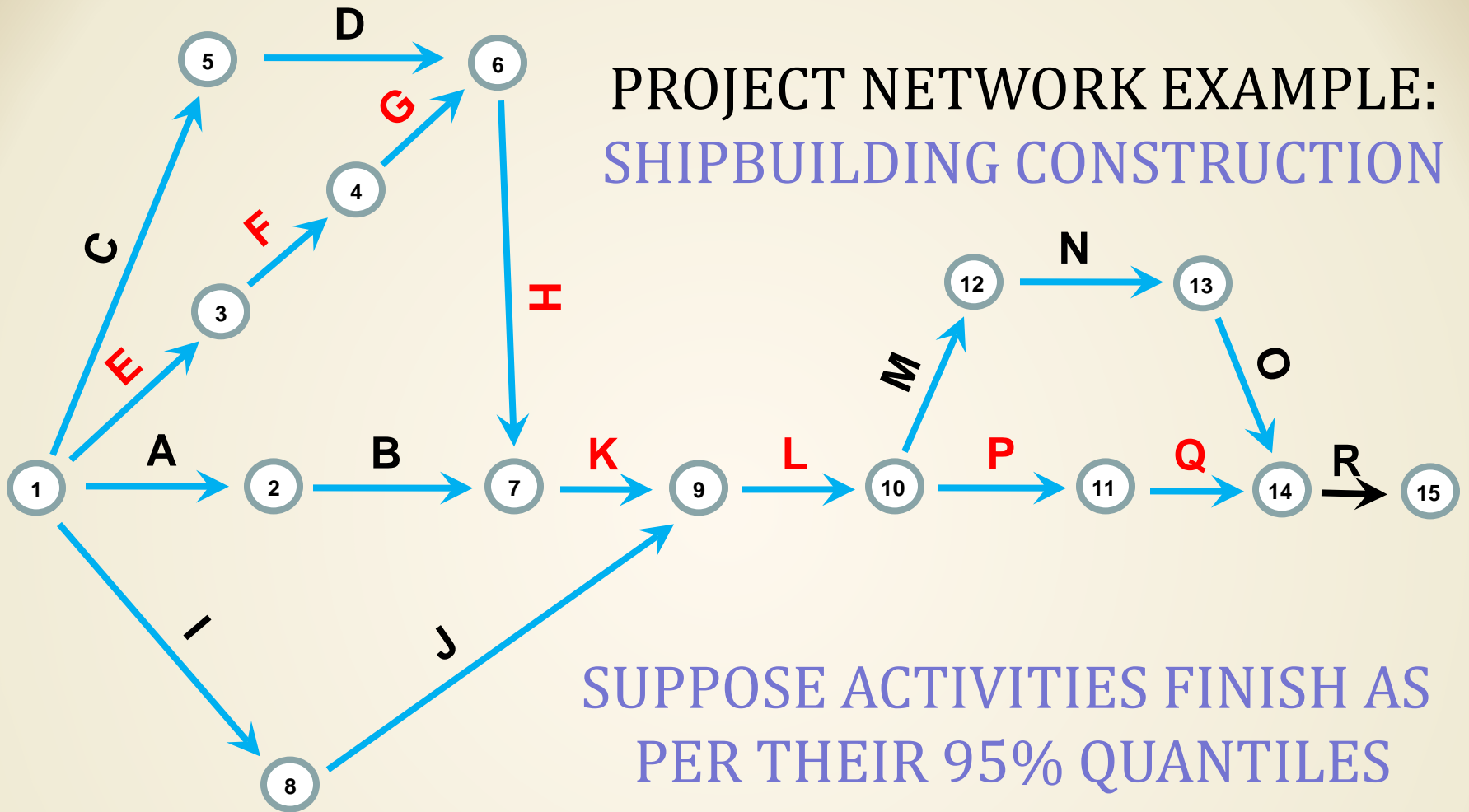


SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES

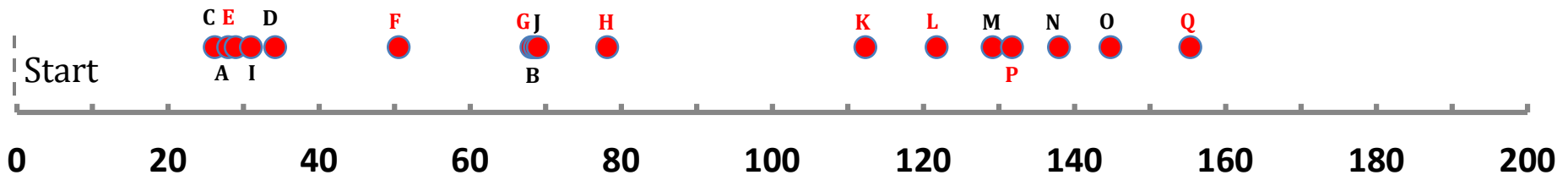




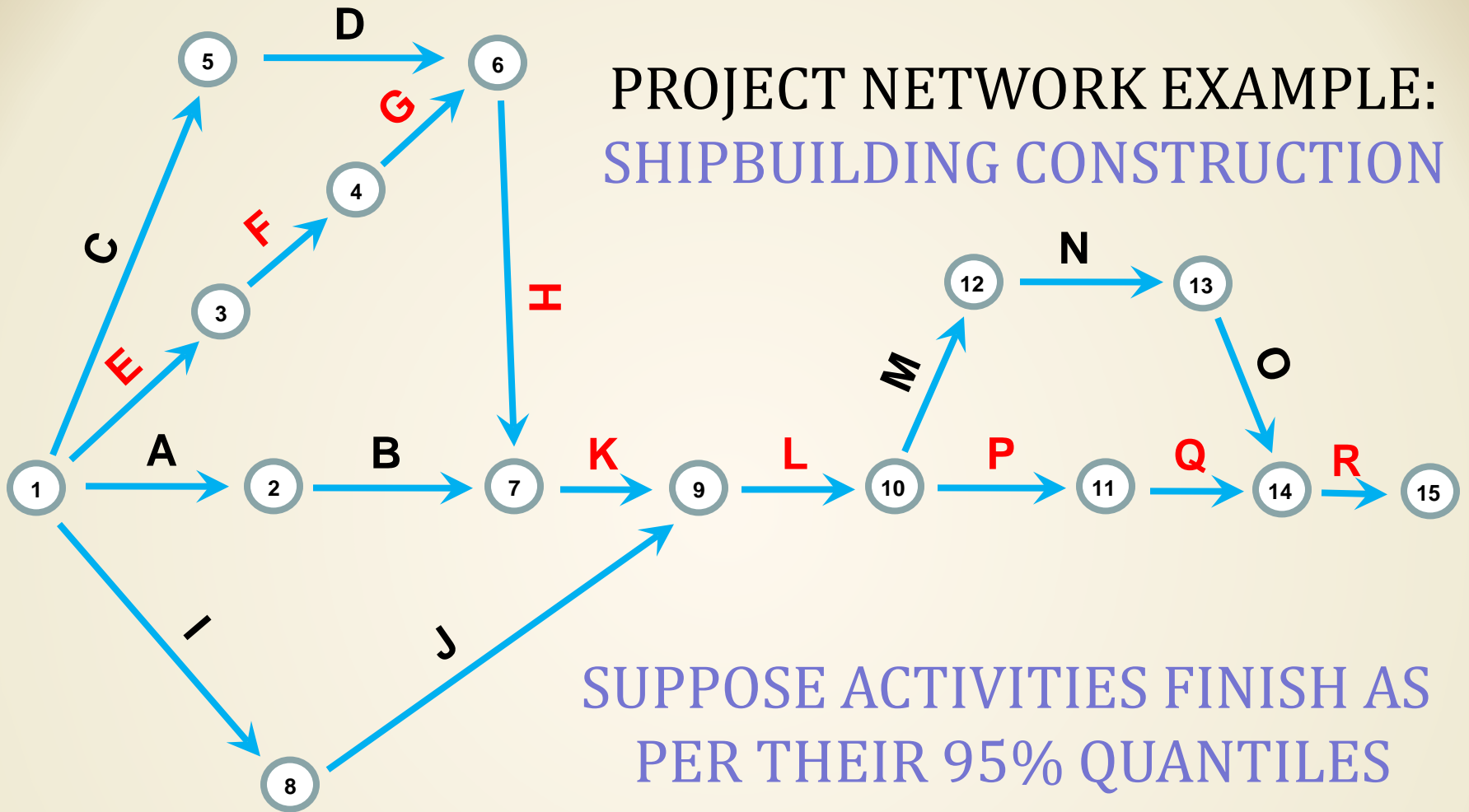
# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



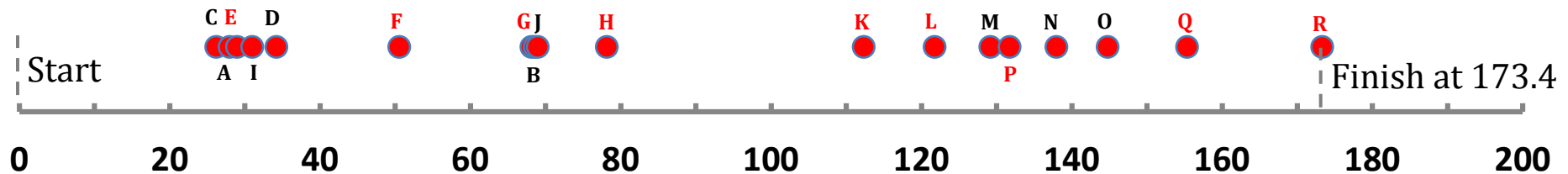
SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



# PROJECT NETWORK EXAMPLE: SHIPBUILDING CONSTRUCTION



SUPPOSE ACTIVITIES FINISH AS  
PER THEIR 95% QUANTILES



Our objective is to re-evaluate/monitor remaining  
**Project Completion Time Uncertainty**  
as activities finish one by one

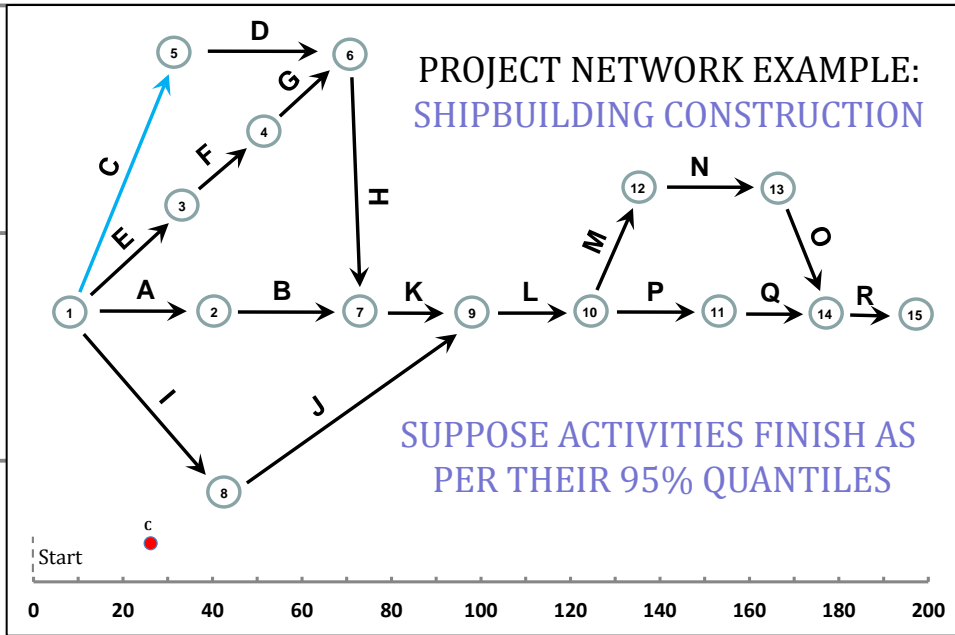
and

To compare the potential implication of neglecting  
**Activity Statistical Dependence** amongst  
remaining activities to be completed.

In this case study average **activity correlation is approximately 0.5**. Effects to be observed would be amplified at higher correlation levels.

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

26 days into the project

A posteriori a difference of  
11 days in projected  
completion time  
"on average"

125 135 145 155 165 175 185 195

152.2

163.3

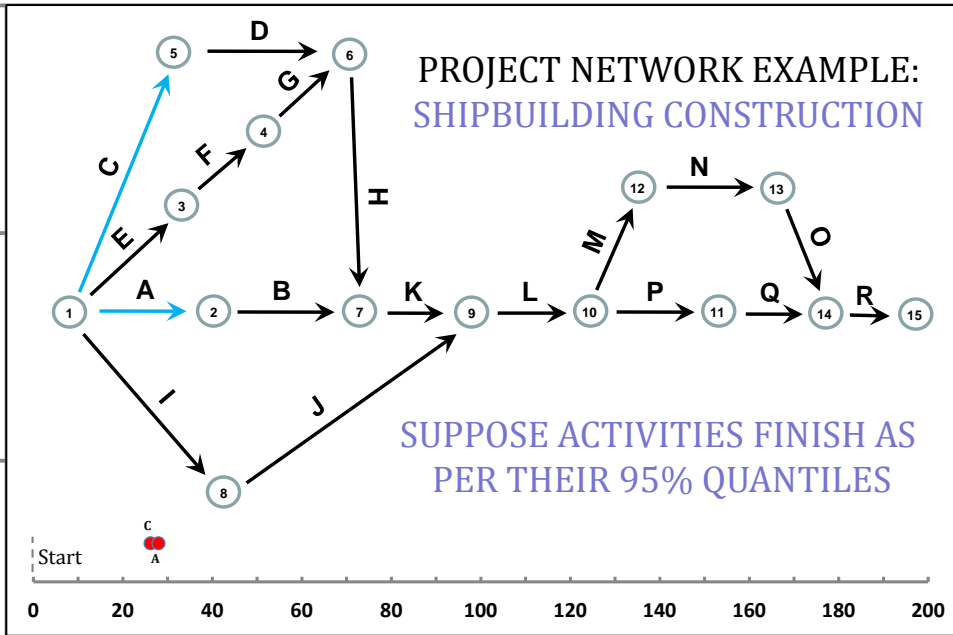
CRITICAL PATH: E F G H K L P Q R

Project Completion Time

— DEP P|C — IND P|C

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

125 135 145 155 165 175 185 195

152.5

167.2

CRITICAL PATH: E F G H K L P Q R

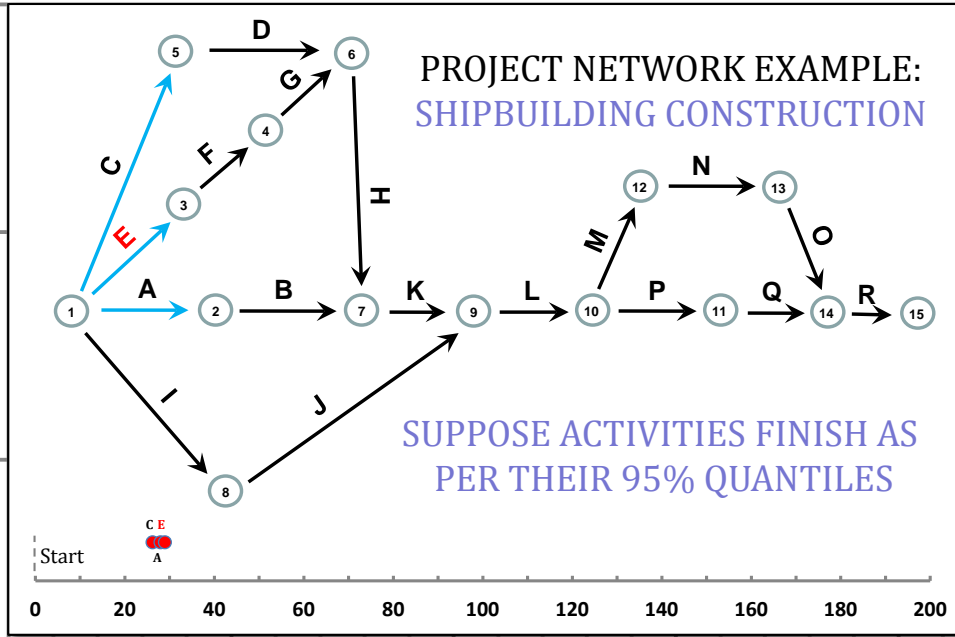
Project Completion Time

— DEP P|CA

— IND P|CA

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

125 135 145 155 165 175 185 195

154.6

169.4

CRITICAL PATH: **E F G H K L P Q R**

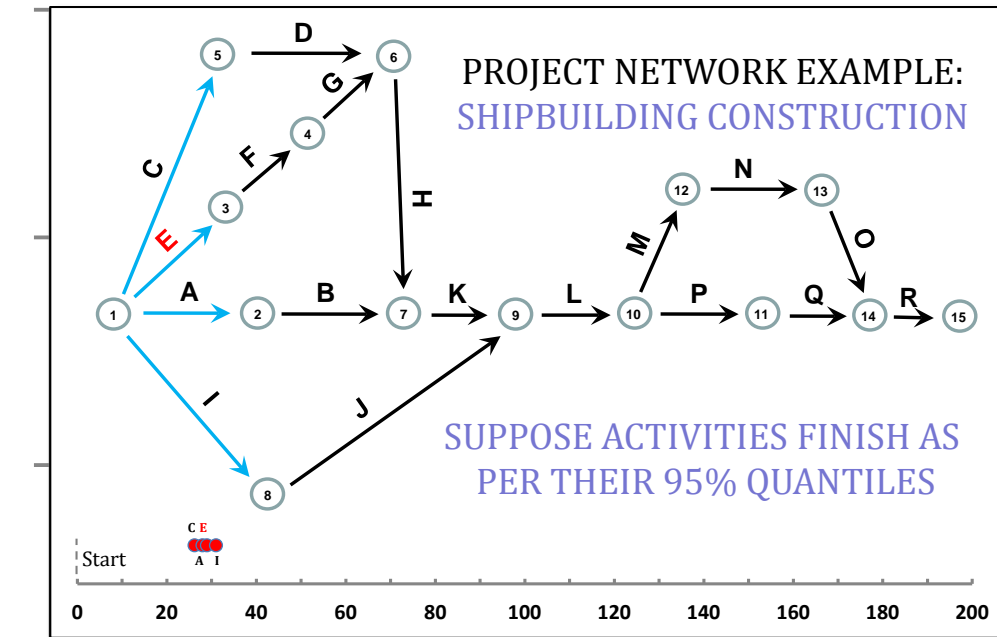
Project Completion Time

— DEP P|CAE

— IND P|CAE

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

32 days into the project

A posteriori a difference of 15 days in projected completion time "on average"

125 135 145 155 165 175 185 195

154.6

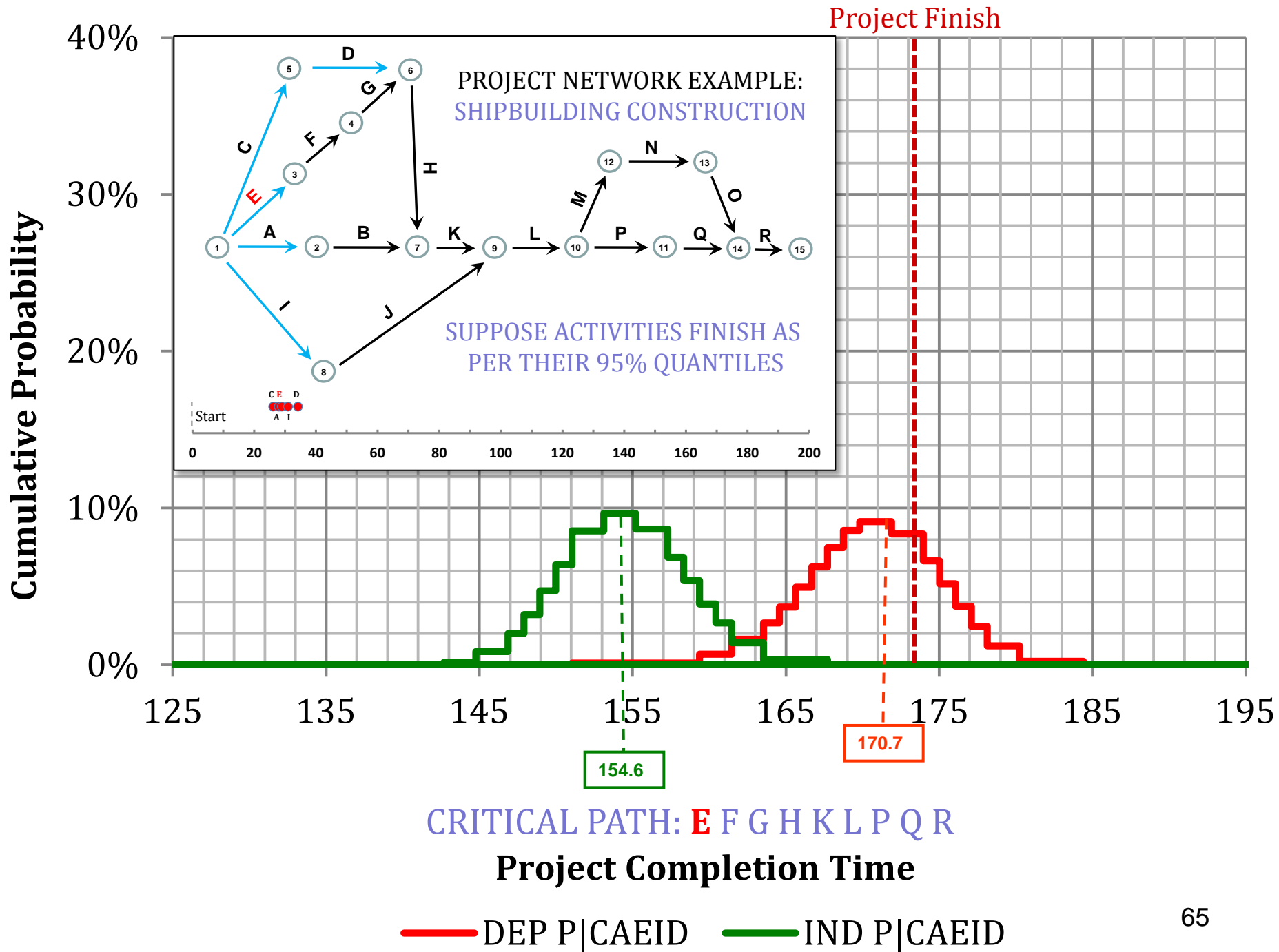
170.3

CRITICAL PATH: E F G H K L P Q R

Project Completion Time

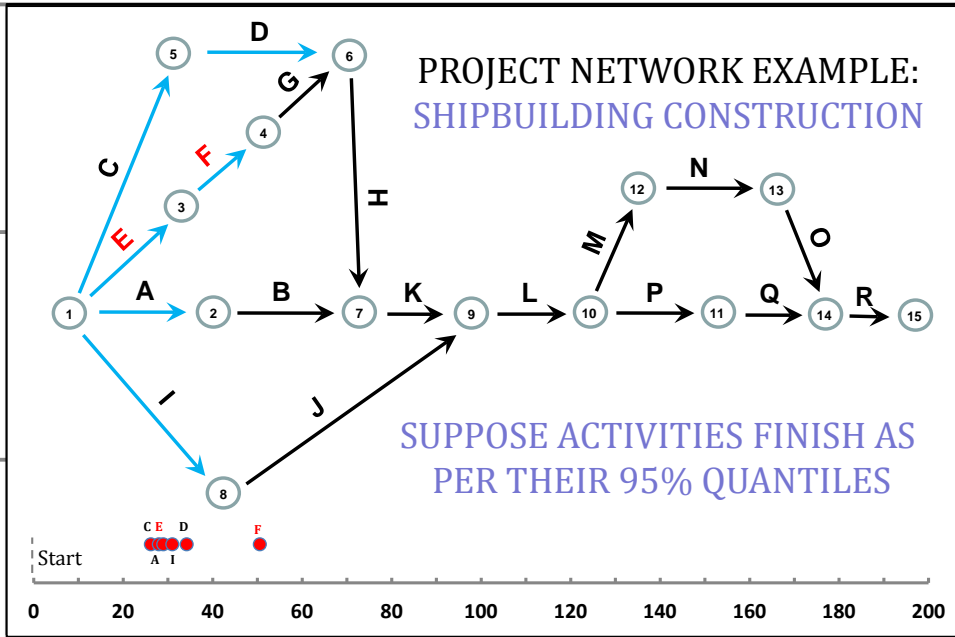
— DEP P|CAEI — IND P|CAEI





Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

50 days into the project

Project Completion Time  
standard deviation  
is about the same !!!

125 135 145 155 165 175 185 195

157.0

171.2

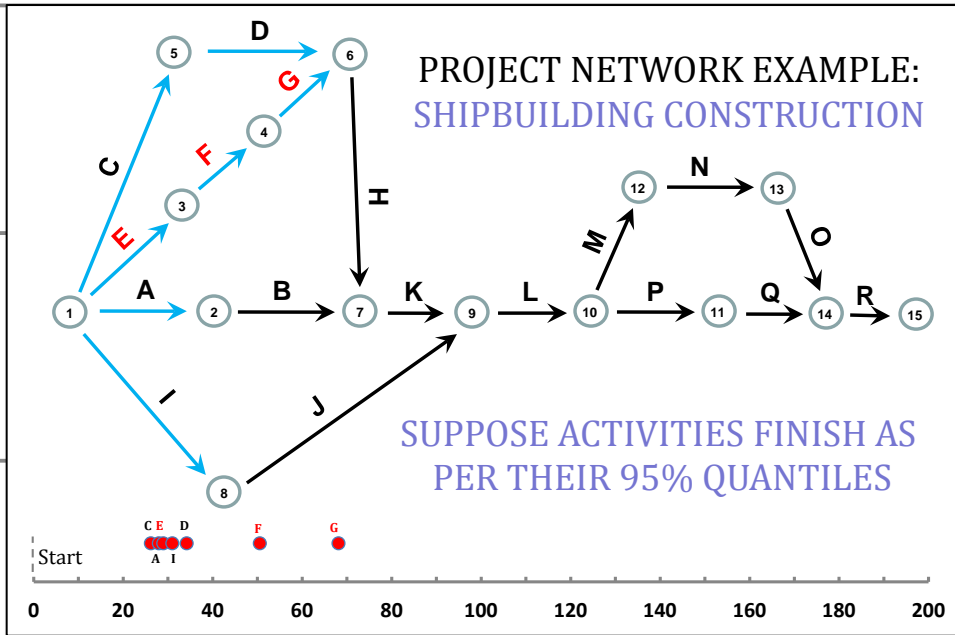
CRITICAL PATH: **E F G H K L P Q R**

Project Completion Time

— DEP P|CAEIDF — IND P|CAEIDF

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

125 135 145 155 165 175 185 195

159.8

171.8

CRITICAL PATH: **E F G H K L P Q R**

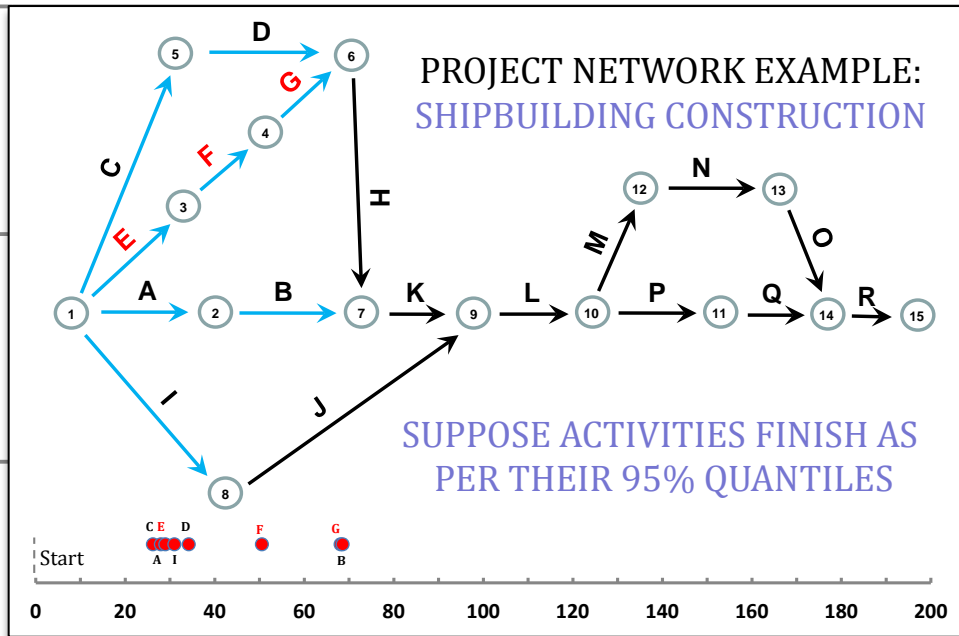
Project Completion Time

— DEP P|CAEIDFG

— IND P|CAEIDFG

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

125 135 145 155 165 175 185 195

159.8

172.0

CRITICAL PATH: **E F G H K L P Q R**

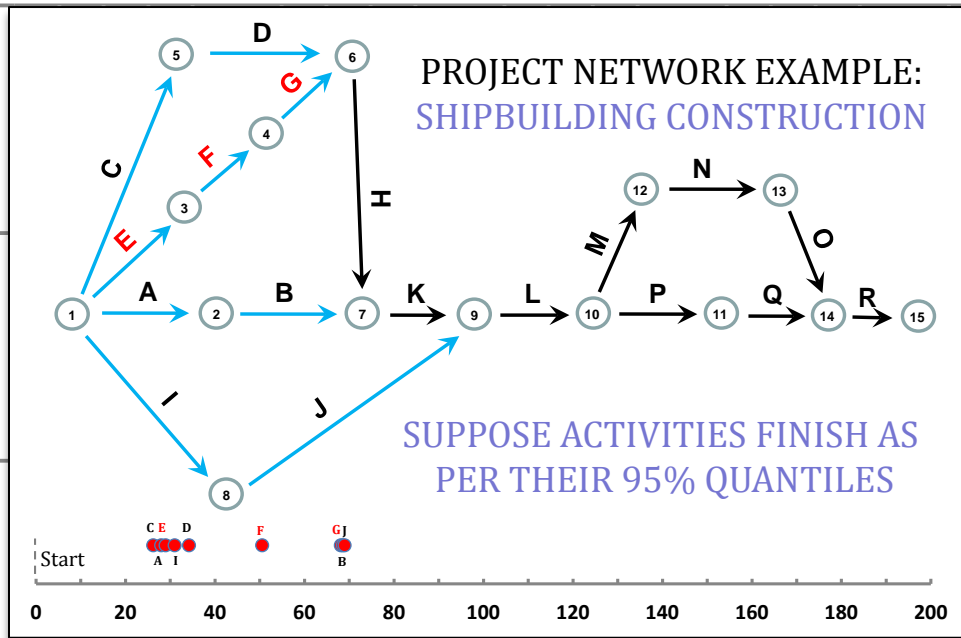
Project Completion Time

— DEP P|CAEIDFGB

— IND P|CAEIDFGB

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

125 135 145 155 165 175 185 195

159.8

172.1

CRITICAL PATH: **E F G H K L P Q R**

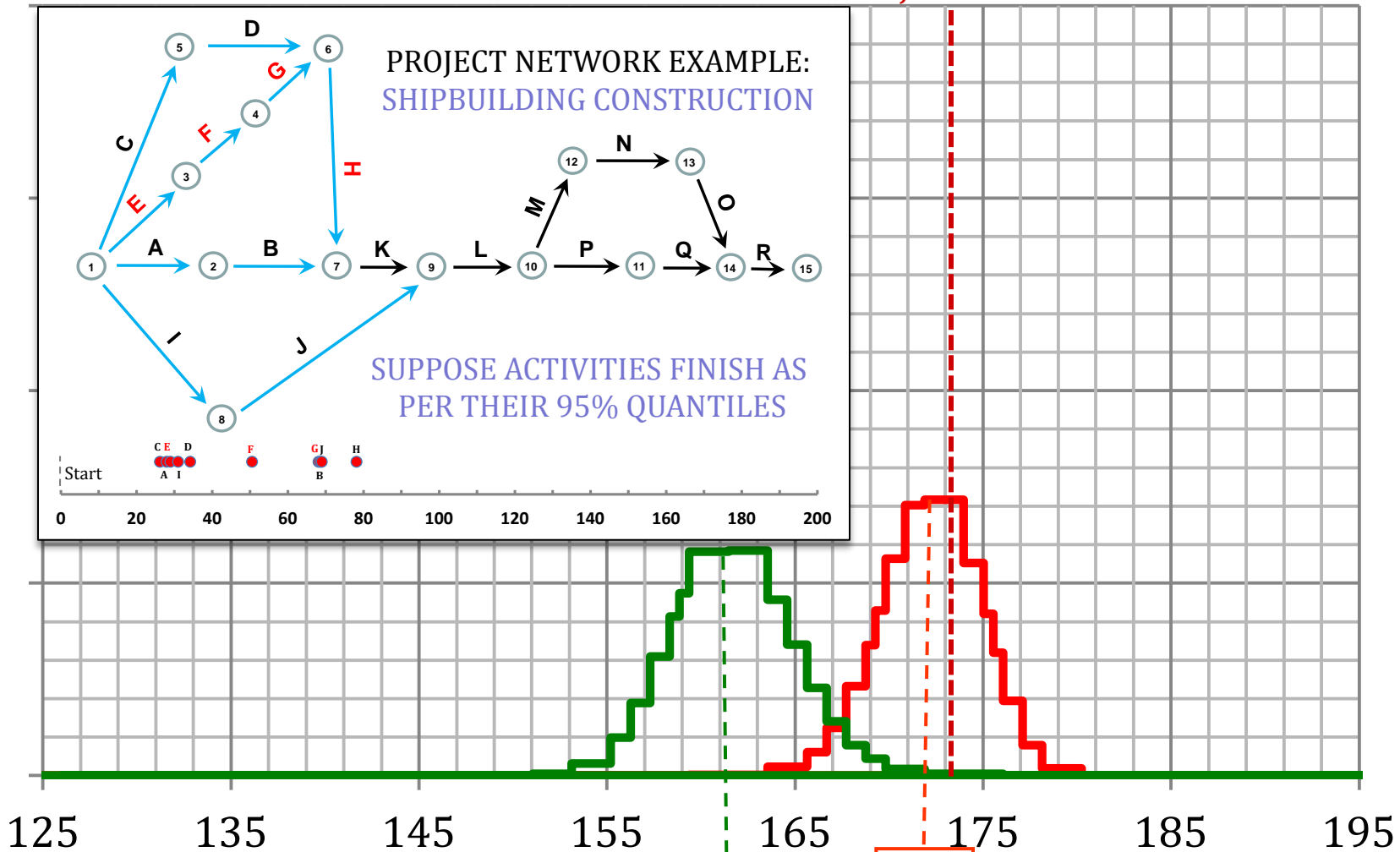
Project Completion Time

— DEP P|CAEIDFGBJ

— IND P|CAEIDFGBJ

Cumulative Probability

40%  
30%  
20%  
10%  
0%

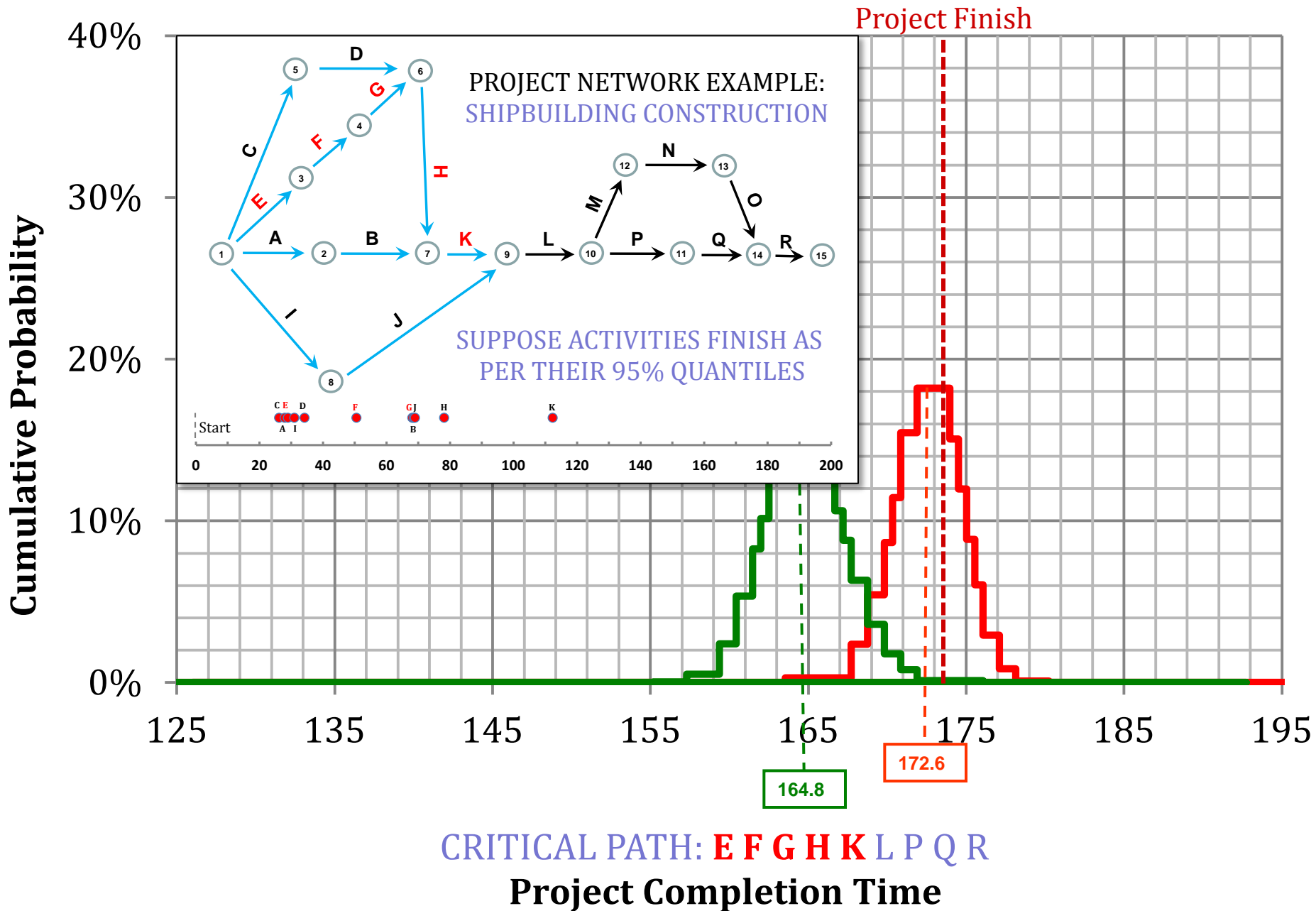


Project Finish

CRITICAL PATH: **E F G H K L P Q R**

Project Completion Time

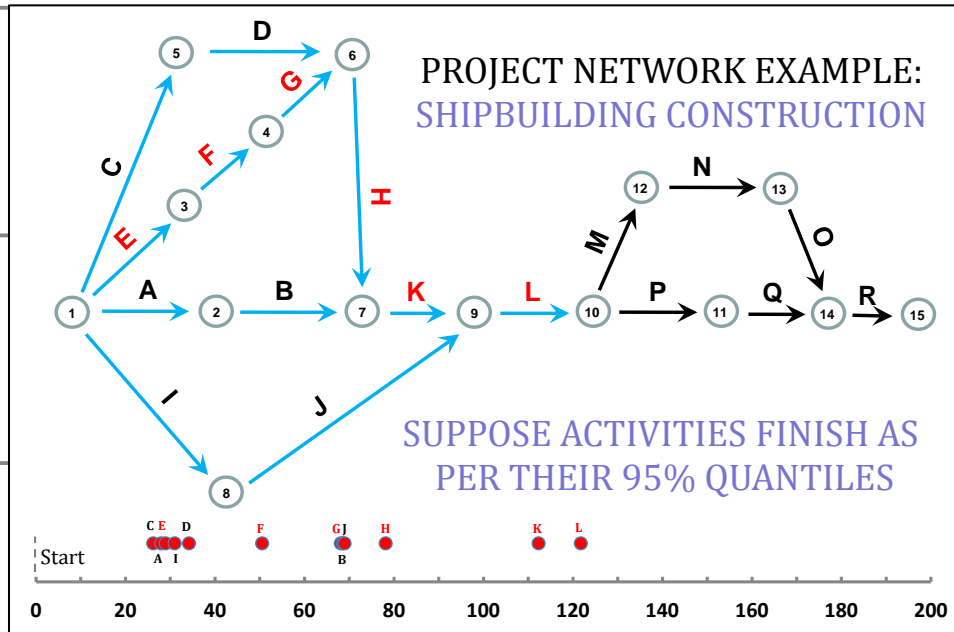
— DEP P|CAEIDFGBJH — IND P|CAEIDFGBJH



— DEP P|CAEIDFGBJHK — IND P|CAEIDFGBJHK

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

121 days into the project

A posteriori a difference of  
6 days in projected  
completion time  
"on average"

Too late to take  
corrective action?

125 135 145 155 165 175 185 195

166.5

172.7

CRITICAL PATH: E F G H K L P Q R

Project Completion Time

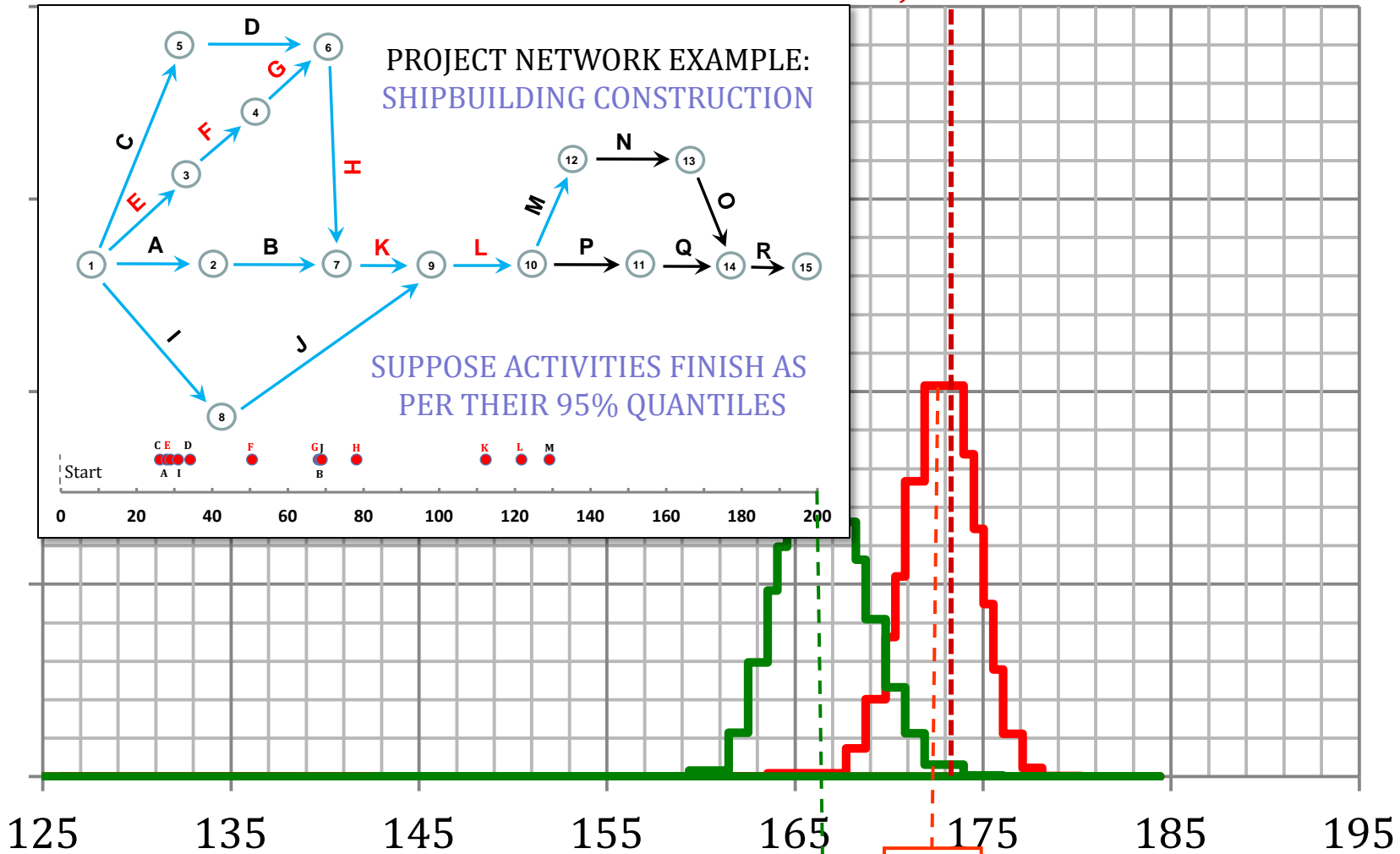
— DEP P|CAEIDFGBJHKL

— IND P|CAEIDFGBJHKL



Cumulative Probability

40%  
30%  
20%  
10%  
0%

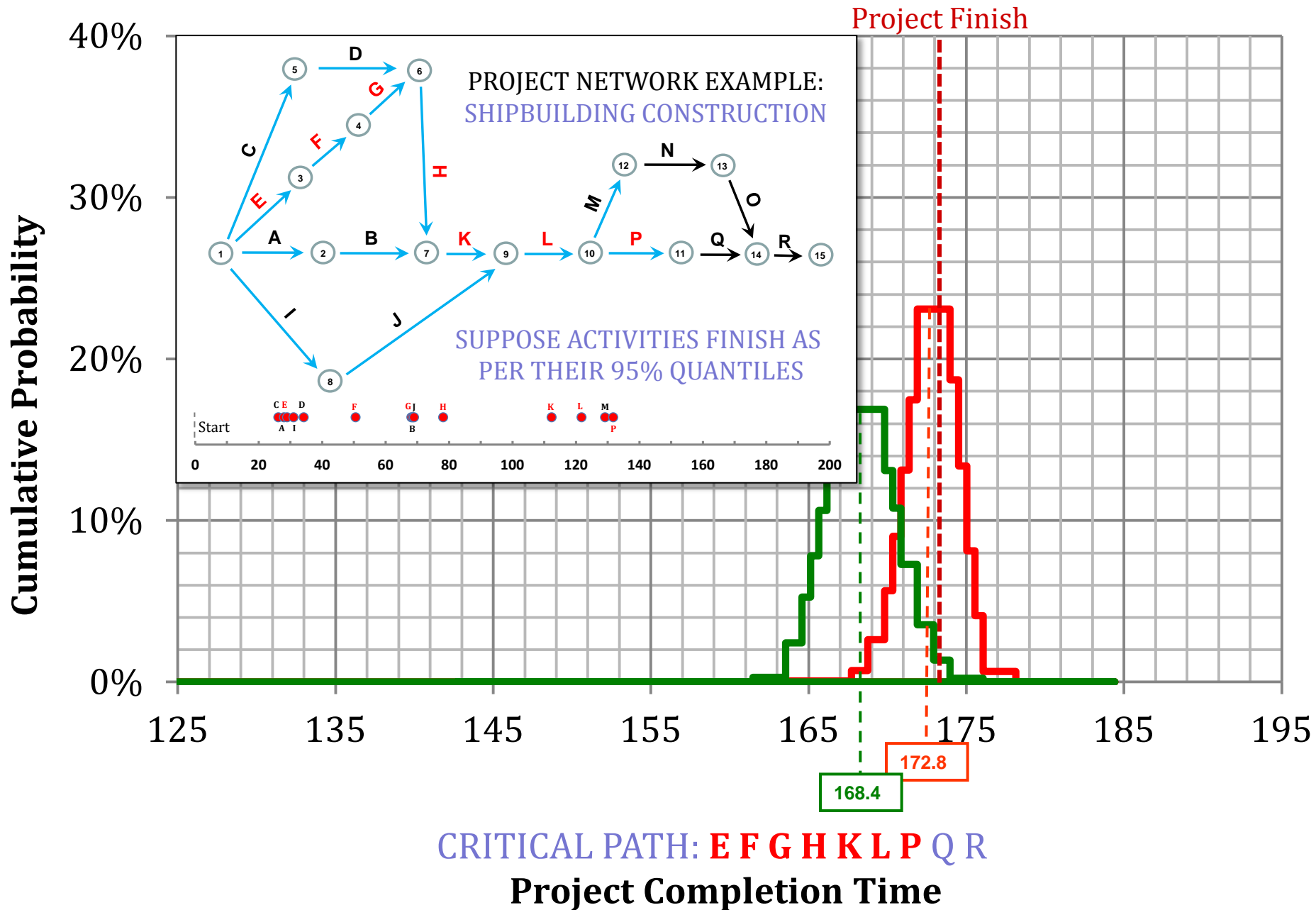


CRITICAL PATH: **E F G H K L P Q R**

Project Completion Time

— DEP P|CAEIDFGBJHKLM

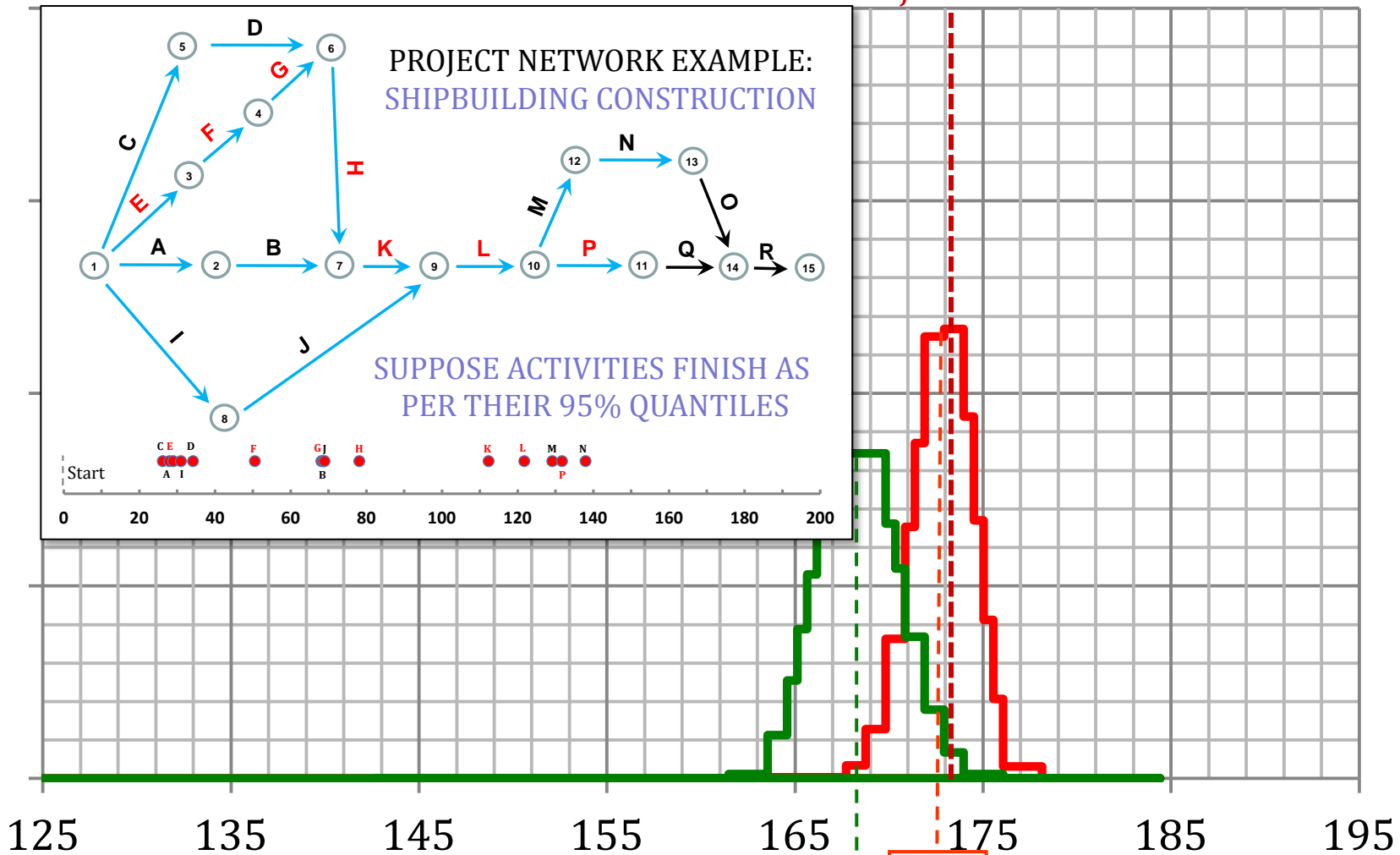
— IND P|CAEIDFGBJHKLM



— DEP P|CAEIDFGBJHKLMP      — IND P|CAEIDFGBJHKLMP<sup>74</sup>

Cumulative Probability

40%  
30%  
20%  
10%  
0%

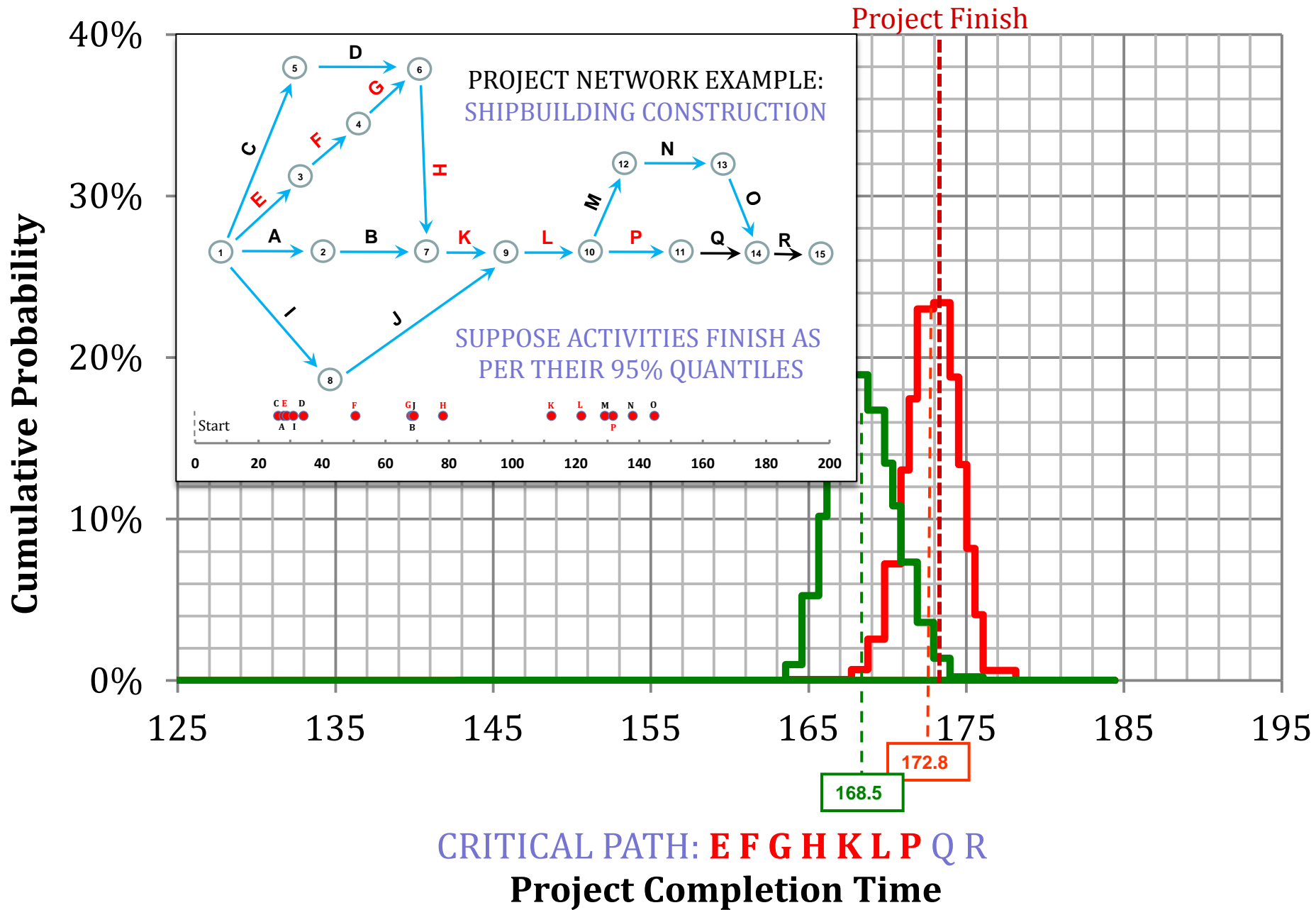


CRITICAL PATH: **E F G H K L P Q R**

Project Completion Time

— DEP P|CAEIDFGBJHKLMPN

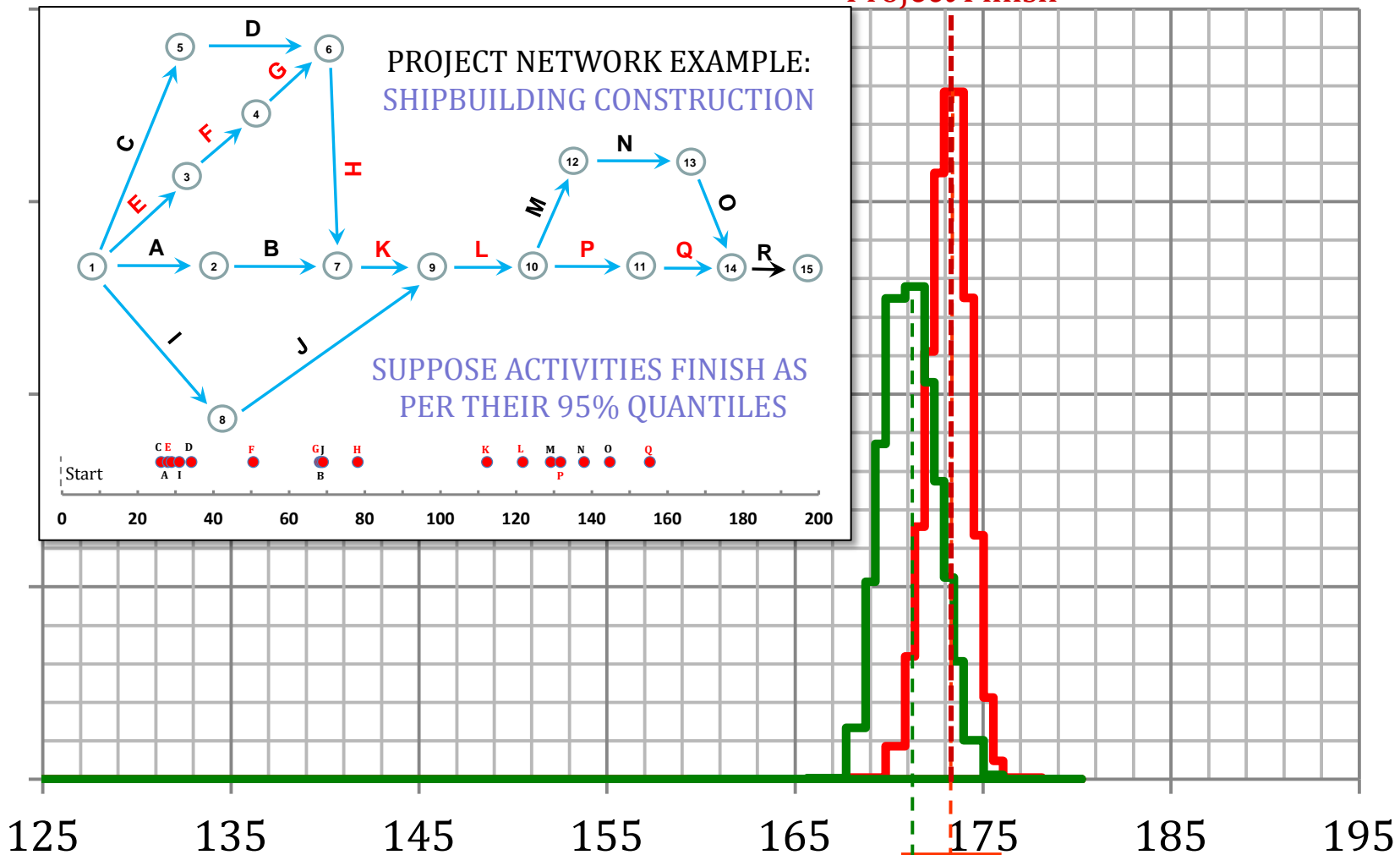
— IND P|CAEIDFGBJHKLMPN



— DEP P|CAEIDFGBJHKLMPNO — IND P|CAEIDFGBJHKLMPNO

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

CRITICAL PATH: **E F G H K L P Q R**

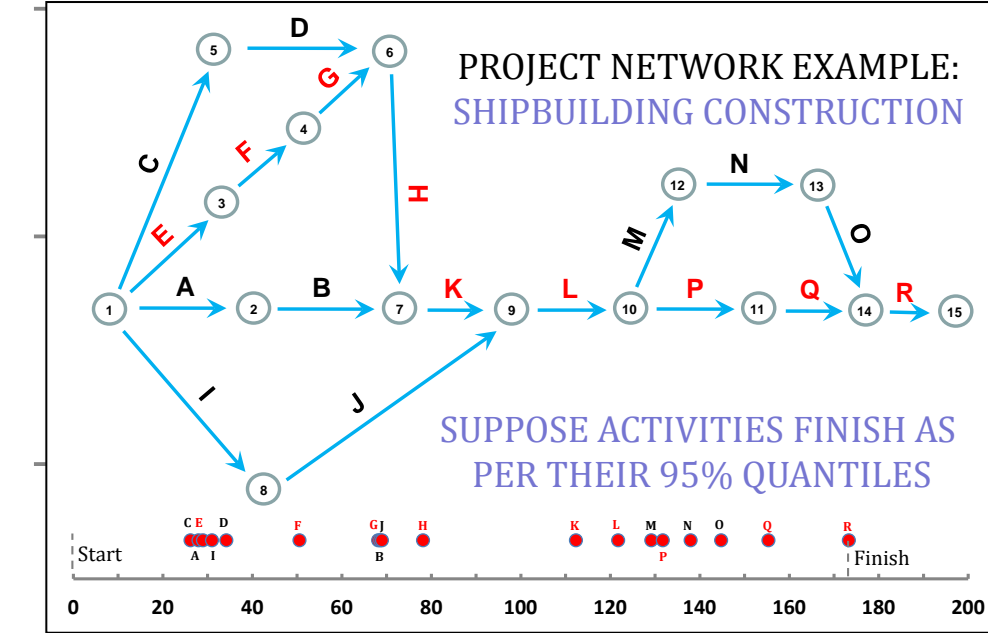
Project Completion Time

— DEP P|CAEIDFGBJHKLMPNOQ

— IND P|CAEIDFGBJHKLMPNOQ

Cumulative Probability

40%  
30%  
20%  
10%  
0%



Project Finish

125 135 145 155 165 175 185 195

173.4  
173.4

CRITICAL PATH: **E F G H K L P Q R**

Project Completion Time

— DEP P|CAEIDFGBJHKLMPNOQR

— IND P|CAEIDFGBJHKLMPNOQR

# THE GEORGE WASHINGTON UNIVERSITY

## WASHINGTON, DC

- Case Study Description
- Bayesian Network Model
- Statistical Dependence Elicitation
- Uncertainty ( $n$ ) and Dependence ( $\beta$ ) Parameter Selection
- Prior Completion Time Uncertainty
- Posterior Analysis: Monitoring Uncertainty
- Conclusion

# Conclusion

1. Accounting for statistical dependence amongst activities results a **priori** in larger uncertainty bands for the project completion time - **Nothing new here!**
2. As activities complete using the Bayes Network analysis the remaining activity completion uncertainty distributions are updated (as if they were known at the start of the project)
3. The speed of learning about the project completion time is enhanced by modeling the activity statistical dependence using a Bayes Network.
4. In case study one third into the project (i.e. a **posteriori**) uncertainty bands are less taking into account activity statistical dependence.
5. The propagation of the completed activities together with updating remaining activity uncertainty distributions through the project and Bayes network structure allows for **more timely corrective actions** from a project management perspective avoiding:

**Potential Supprises!!!!**



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# THE GEORGE WASHINGTON UNIVERSITY

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WASHINGTON, DC

# QUESTIONS?

Johan Rene van Dorp  
and Ifechukwu Nduka

**SAN FRANCISCO 2014**  
INFORMS ANNUAL MEETING  
BRIDGING DATA AND DECISIONS



THE GEORGE WASHINGTON UNIVERSITY  
SCHOOL OF ENGINEERING  
AND APPLIED SCIENCE